Estimation over the collision channel with private and common observations

Marcos M. Vasconcelos

mvasconc@usc.edu

Dept. of Electrical Engineering

University of Southern California

Nuno C. Martins

nmartins@umd.edu

Dept. of Electrical and Computer Engineering

University of Maryland, College Park

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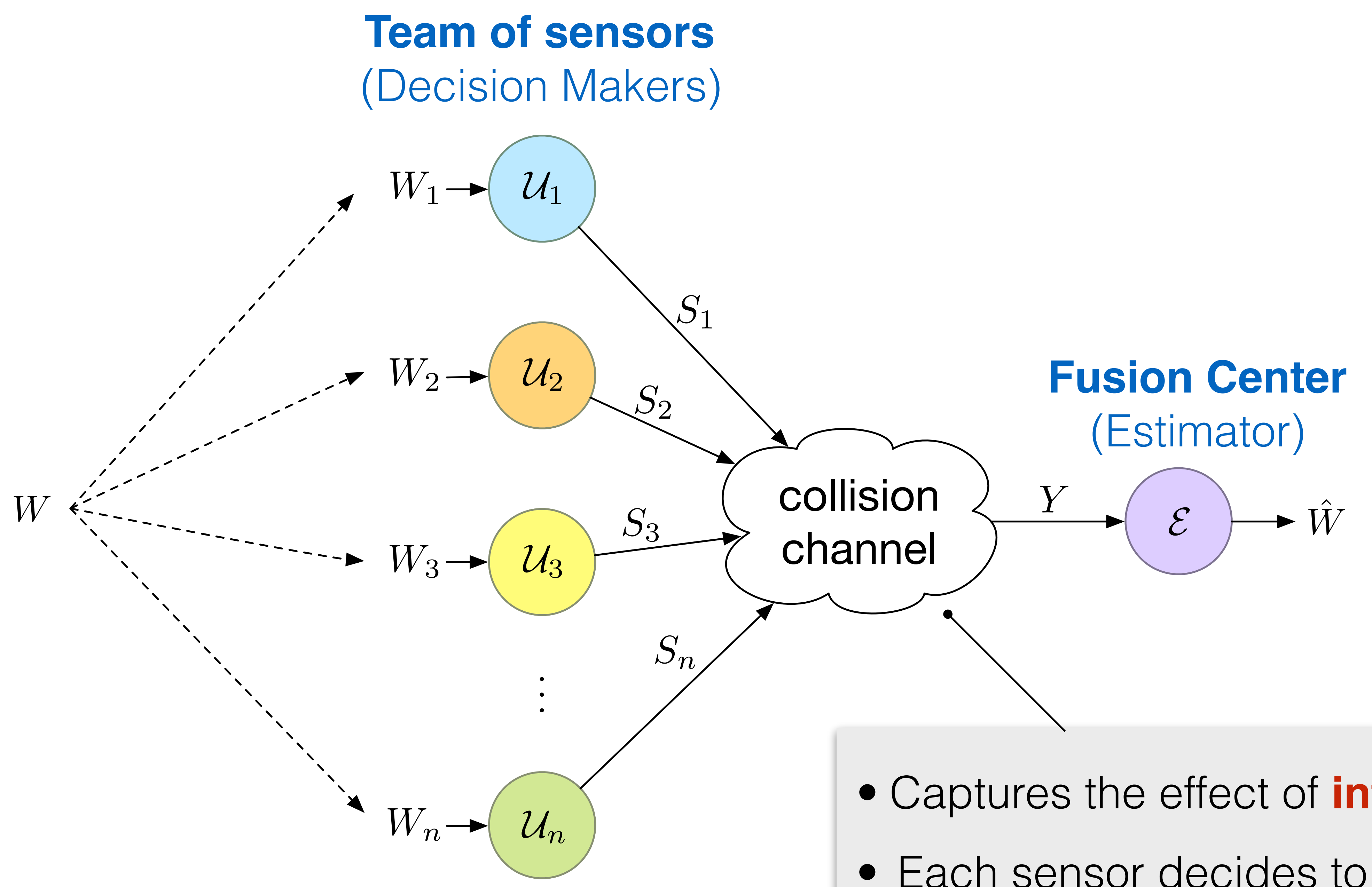
Basic framework

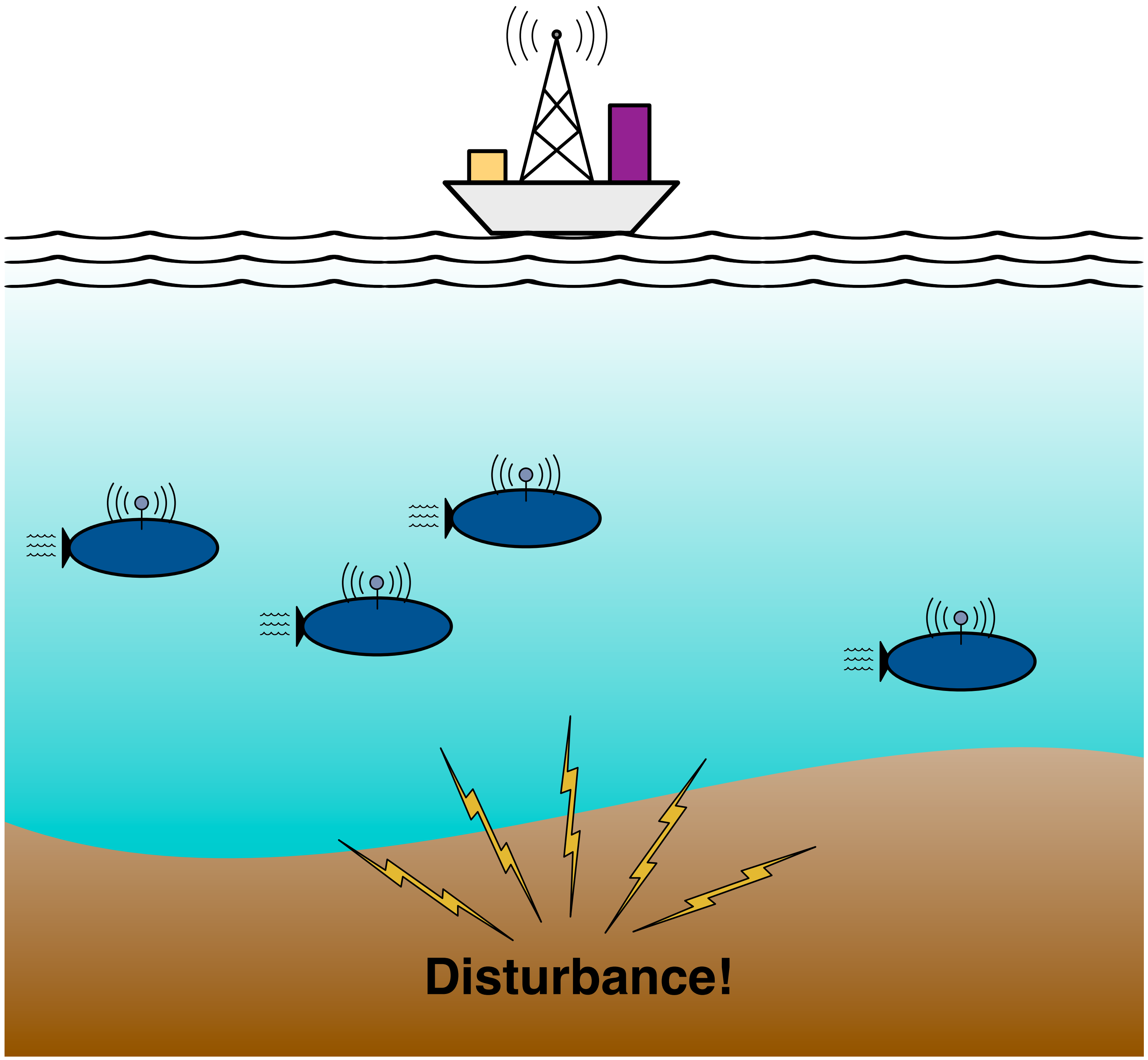


- Captures the effect of **interference**
- Each sensor decides to transmit or not
- >1 transmission result in a **collision**

Design jointly optimal communication and estimation policies

Application: Underwater acoustic sensor networks

Environmental monitoring - **quickly** detect a random event or disturbance



Features

- Teams of sensors
- Cooperation
- Decentralized system

Challenges[1,2]

- Collisions (interference)
- Long delays
- Lack of feedback

No coordination protocols

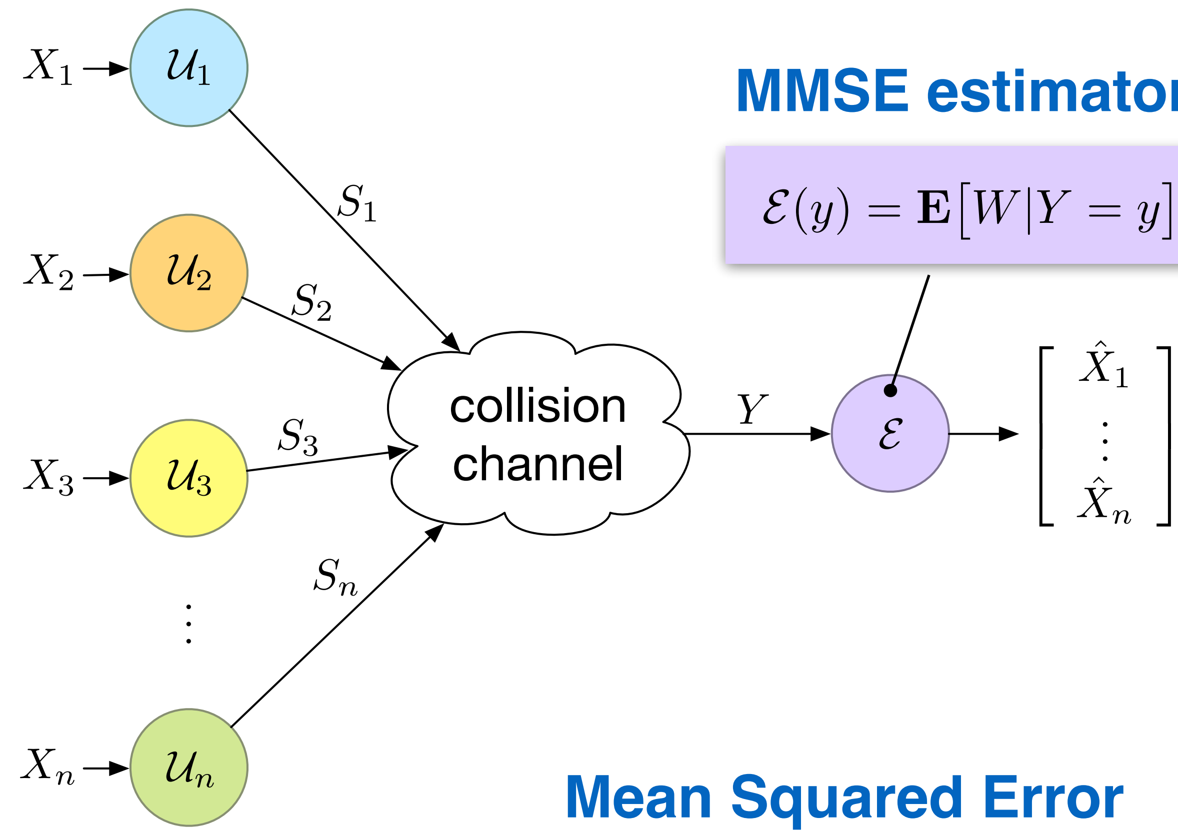
1. Bullo, Cortés and Martínez, *Distributed Control of Robotic Networks*, 2009.
2. Climent et al., "Underwater Acoustic Wireless Sensor Networks," *IEEE Sensors* 2014.

Previous work: MMSE estimation over the collision channel

$$W = [X_1, \cdots, X_n]$$

$$X_i, \quad i \in \{1, \cdots, n\}$$

- mutually **independent**
- **continuous** rvs
- supported on the real line
- **any distribution**



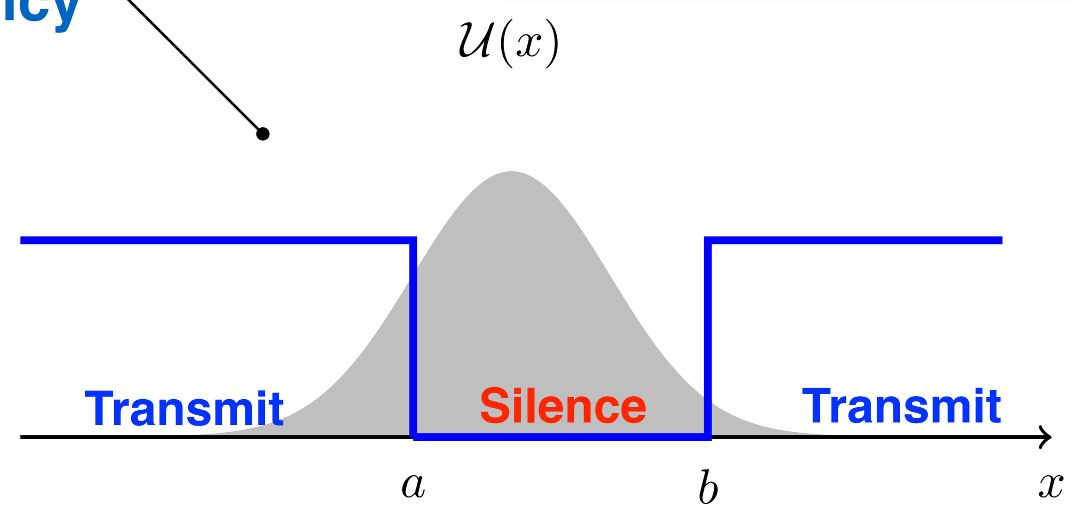
MMSE estimator

$$\mathcal{E}(y) = \mathbf{E}[W|Y=y]$$

Mean Squared Error

$$\text{minimize} \quad \mathcal{J}(\mathcal{U}_1, \ldots, \mathcal{U}_n) = \mathbf{E}\left[\sum_{i=1}^{n} (X_i - \hat{X}_i)^2\right]$$

Threshold policy

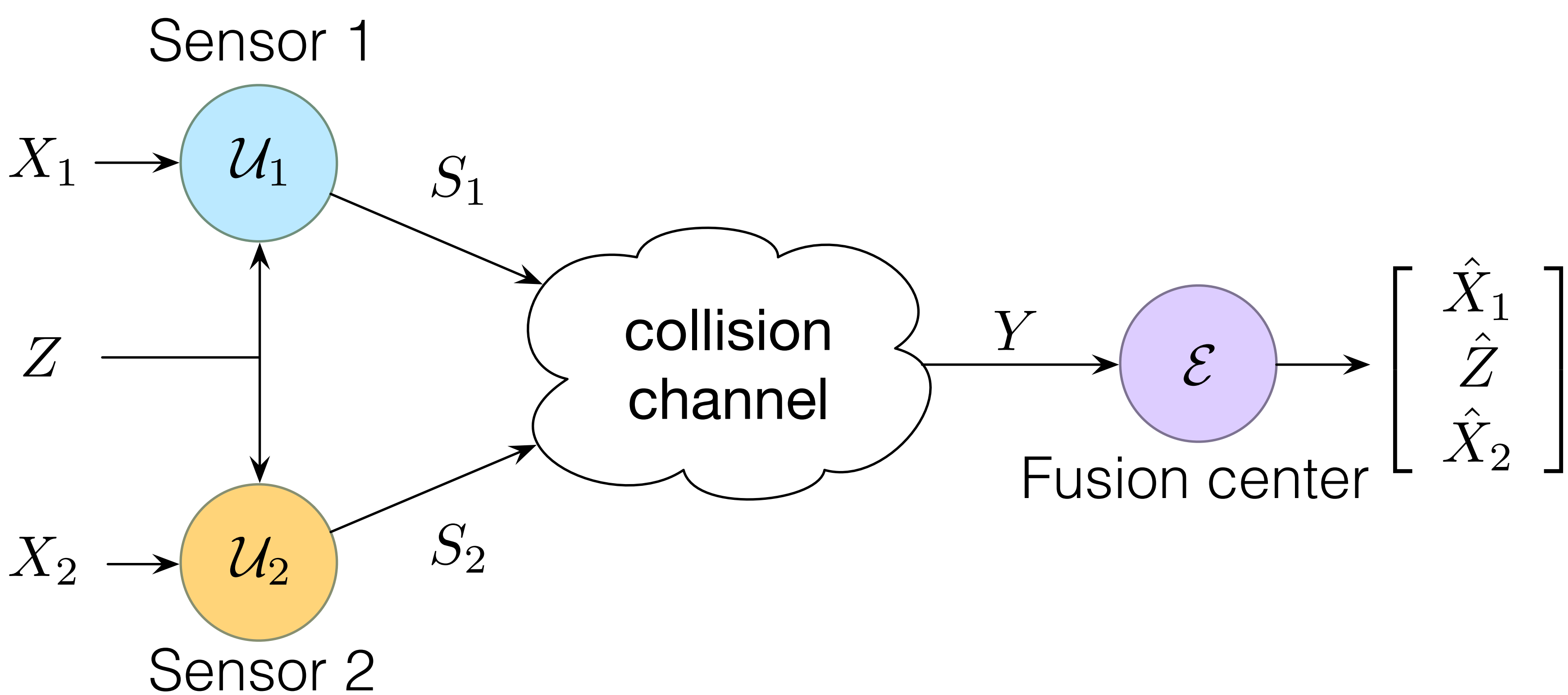


Result[1]

Existence of jointly optimal threshold policies

1. Vasconcelos & Martins, "Optimal estimation over the collision channel," to appear in *IEEE TAC* 2017.

Collision channel with common and private observations



Decision variables: U_i

$$U_i = 1 \implies S_i = (i, Z, X_i)$$

(**transmit**)

$$U_i = 0 \implies S_i = \varnothing$$

(**stay silent**)

Communication policies: $\mathcal{U}_i$

$$\mathcal{U}_i(x,z) = \mathbf{prob}\big(U_i = 1 | X_i = x, Z = z\big)$$

Common and private observations

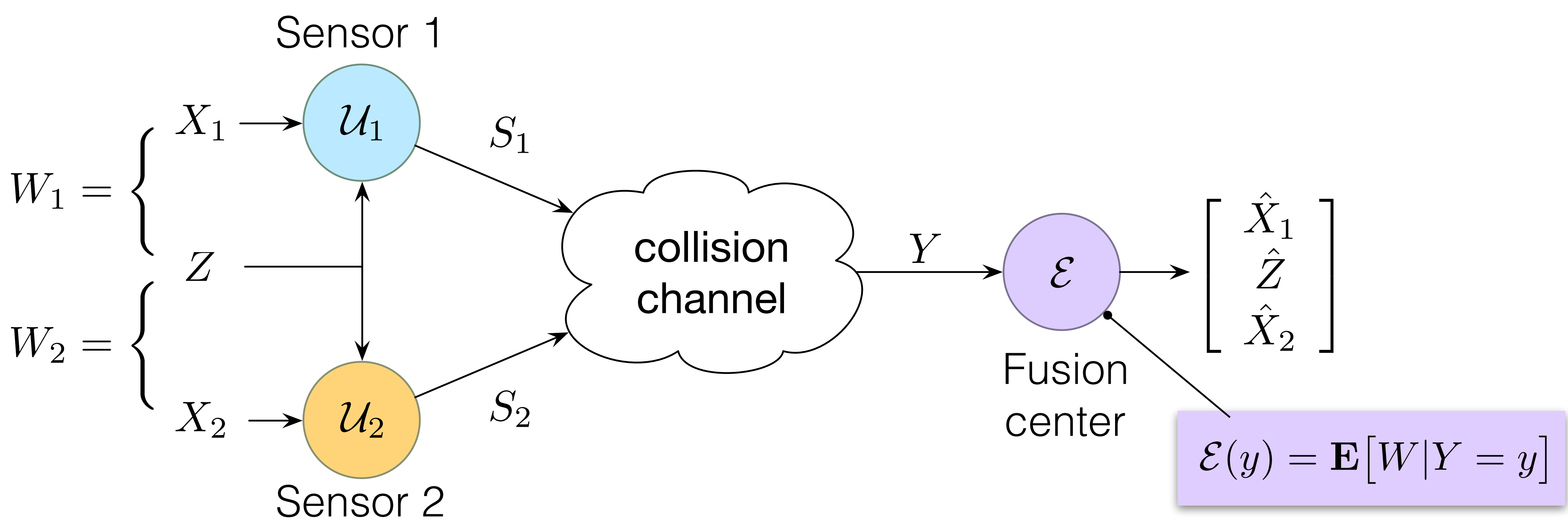
$$W = \begin{bmatrix} X_1 \\ Z \\ X_2 \end{bmatrix} \qquad W_i = \begin{bmatrix} X_i \\ Z \end{bmatrix}$$

private observation (X_i)

common observation (Z)

$$f_W = f_Z \cdot f_{X_1|Z} \cdot f_{X_2|Z}$$

$$X_1 \leftrightarrow Z \leftrightarrow X_2$$



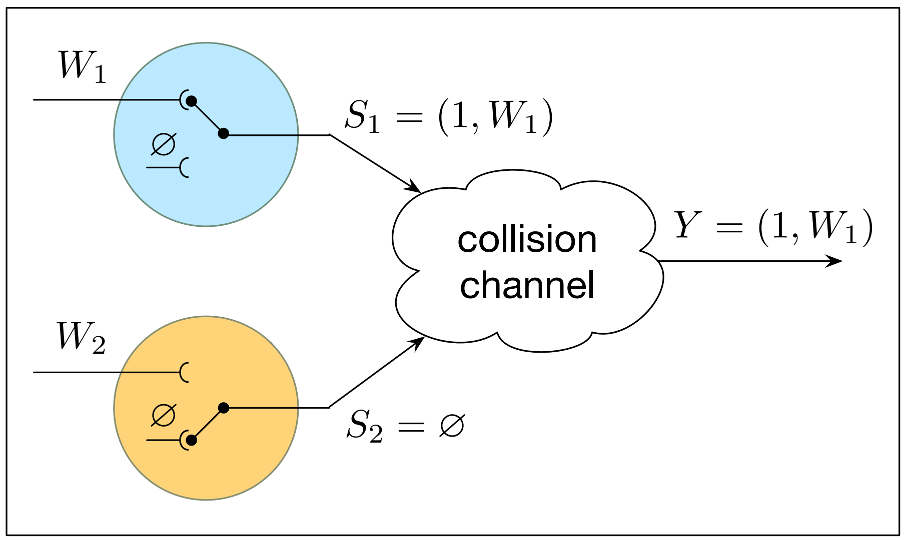
Problem

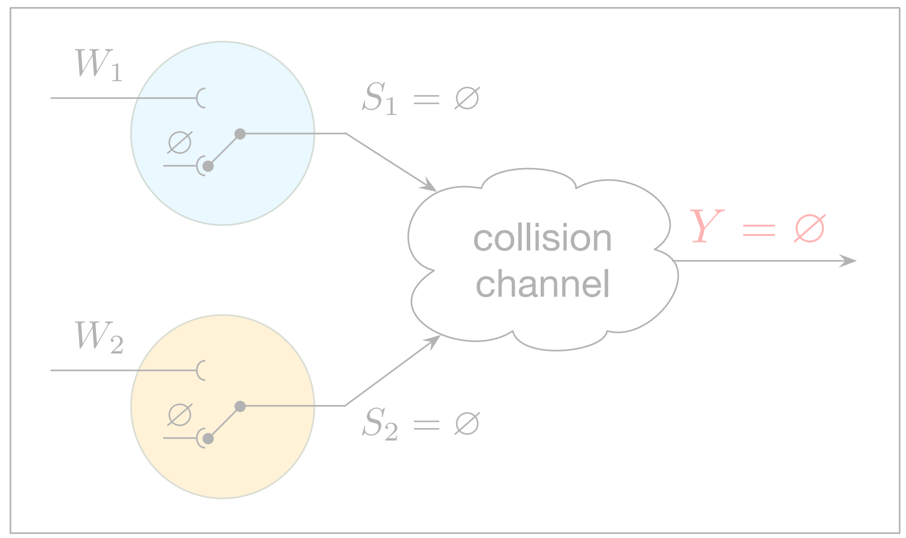
$$\text{minimize} \qquad \mathcal{J}(\mathcal{U}_1,\mathcal{U}_2) = \mathbf{E}\Big[(X_1-\hat{X}_1)^2 + (Z-\hat{Z})^2 + (X_2-\hat{X}_2)^2\Big]$$

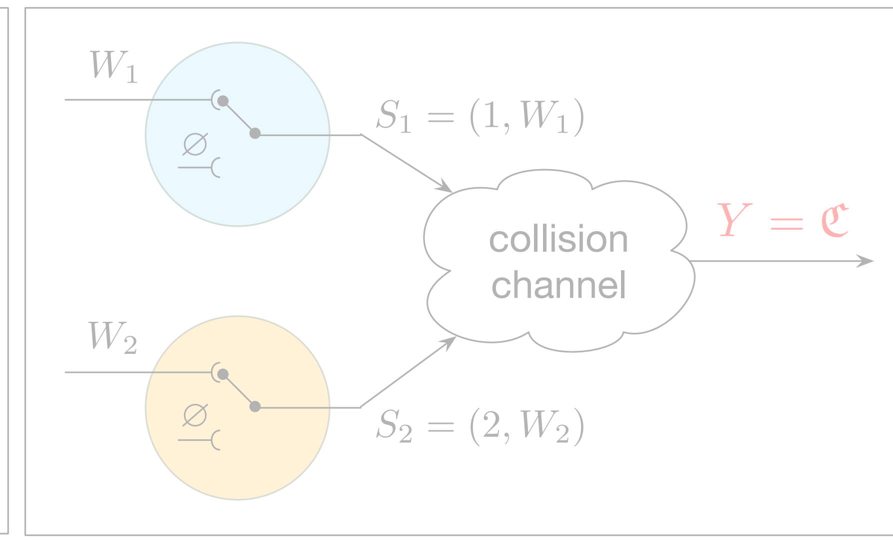
1. van Schuppen, "Common, correlated and private information in control of decentralized systems," Springer 2015.
2. Mahajan, "Optimal decentralized control of coupled subsystems with control sharing", *IEEE TAC* 2013.

Collision channel

single transmission	no transmissions	>1 transmissions
$U_1 = 1, U_2 = 0$	$U_1 = 0, U_2 = 0$	$U_1 = 1, U_2 = 1$







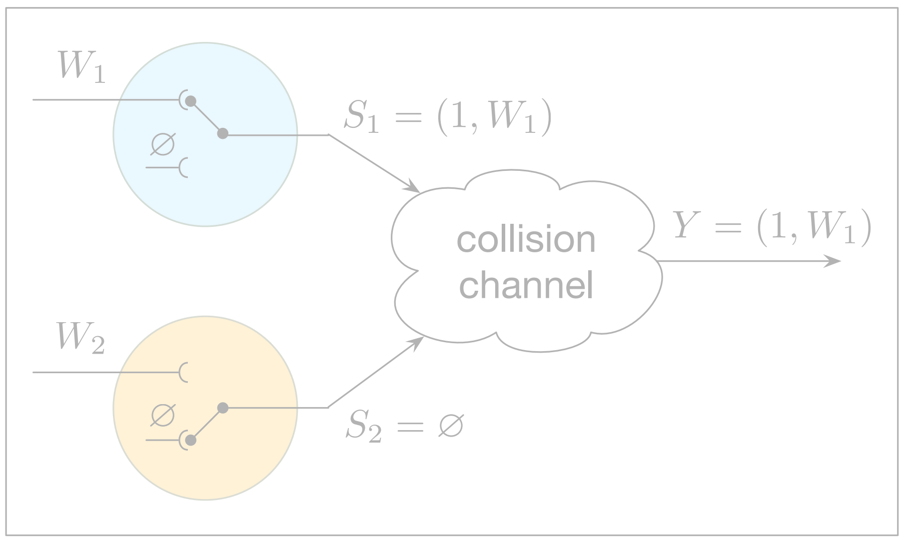
success!	no transmission $\varnothing$	collision $\mathfrak{C}$

From the channel output we can always recover U_1 and U_2.

Collision channel

single transmission

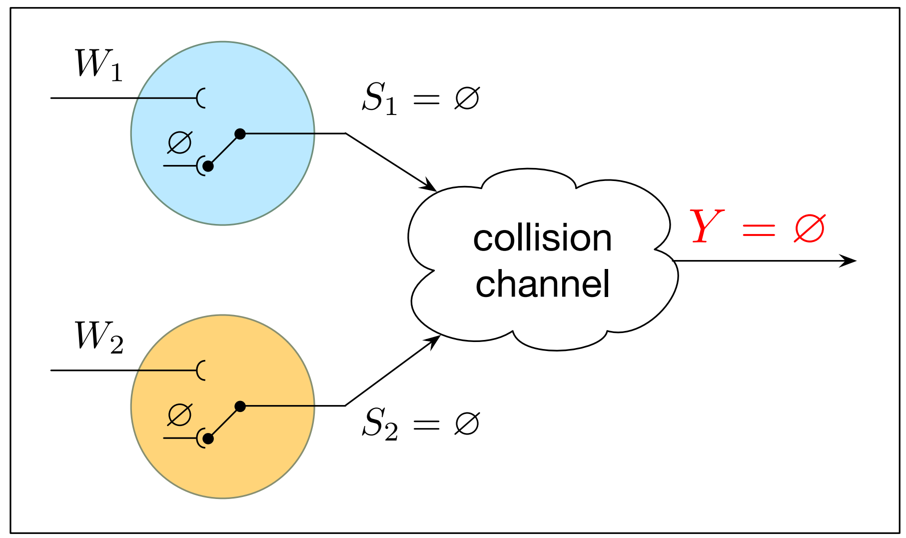
$U_1 = 1, U_2 = 0$



success!

no transmissions

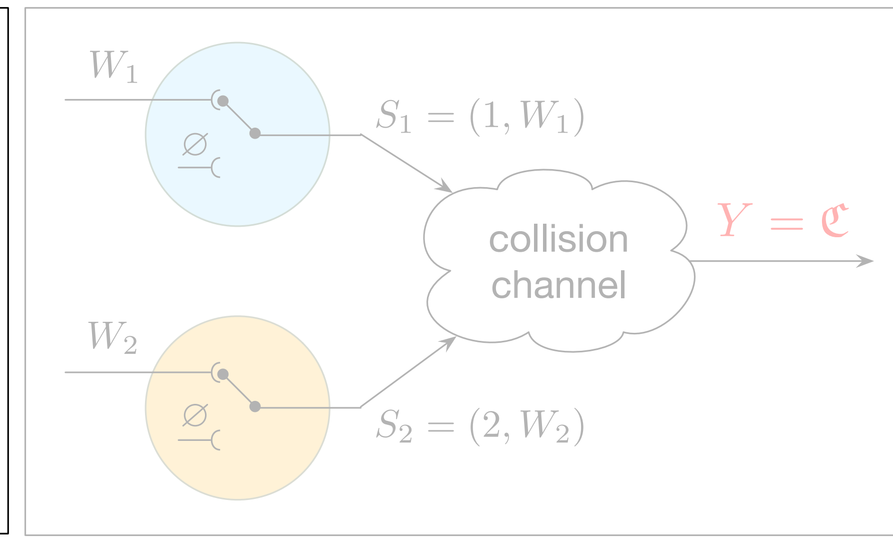
$U_1 = 0, U_2 = 0$



no transmission $\varnothing$

>1 transmissions

$U_1 = 1, U_2 = 1$

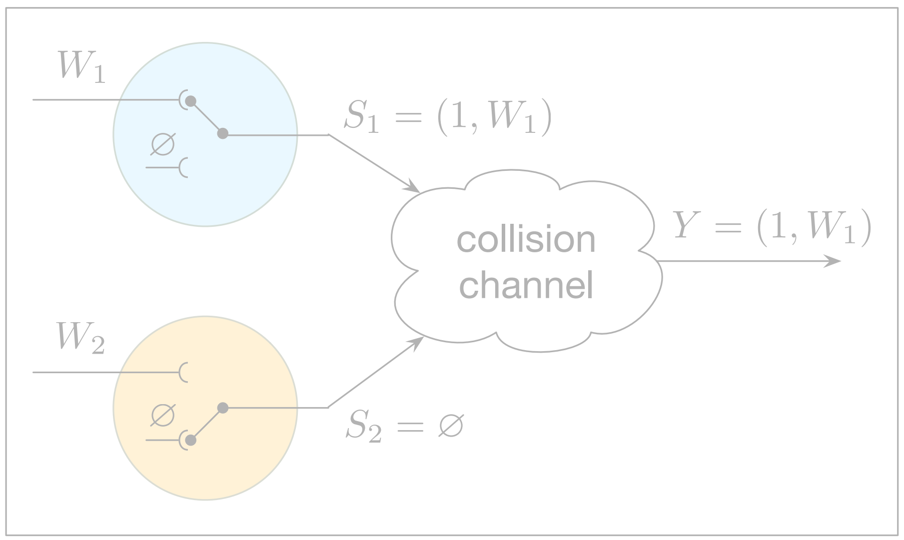


collision $\mathfrak{C}$

From the channel output we can always recover U_1 and U_2.

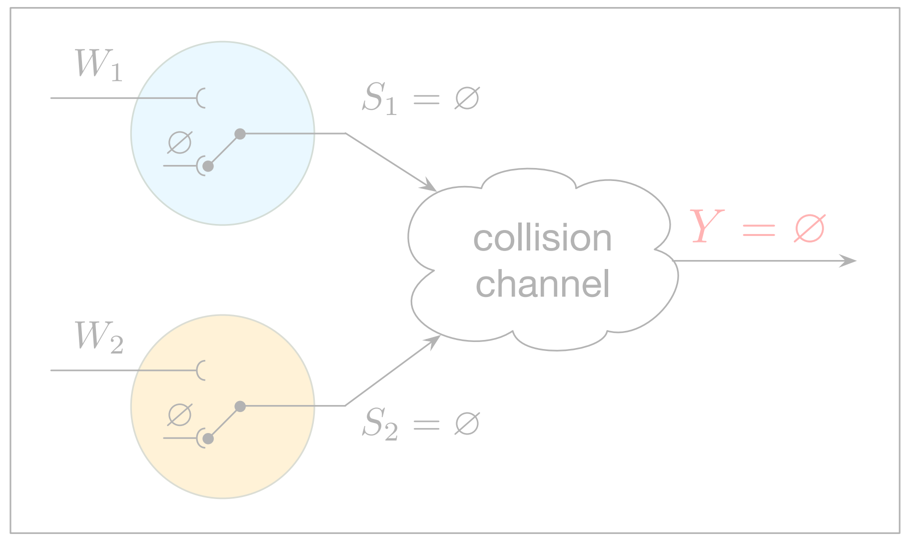
Collision channel

single transmission
$U_1 = 1, U_2 = 0$



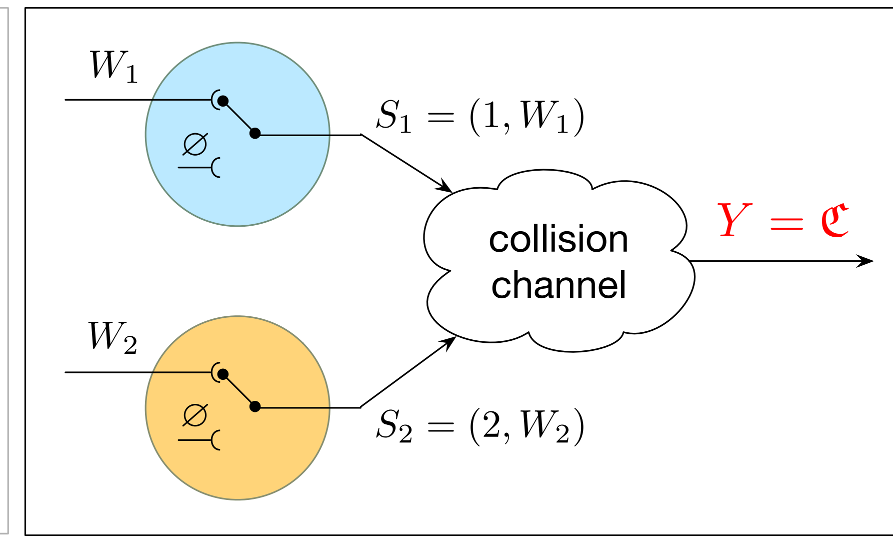
success!

no transmissions
$U_1 = 0, U_2 = 0$



no transmission $\varnothing$

>1 transmissions
$U_1 = 1, U_2 = 1$



collision $\mathfrak{C}$

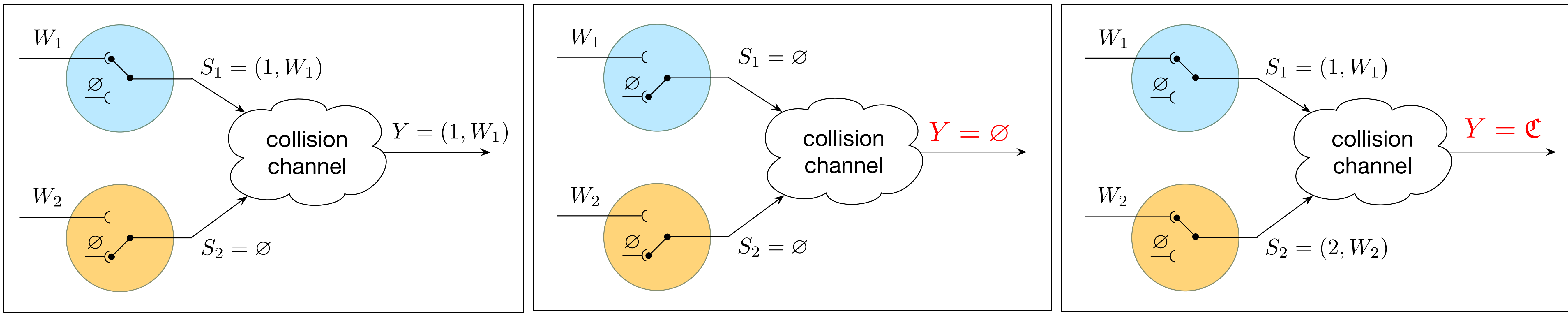
From the channel output we can always recover U_1 and U_2.

Collision channel

single transmission $U_1 = 1, U_2 = 0$

no transmissions $U_1 = 0, U_2 = 0$

>1 transmissions $U_1 = 1, U_2 = 1$



success!

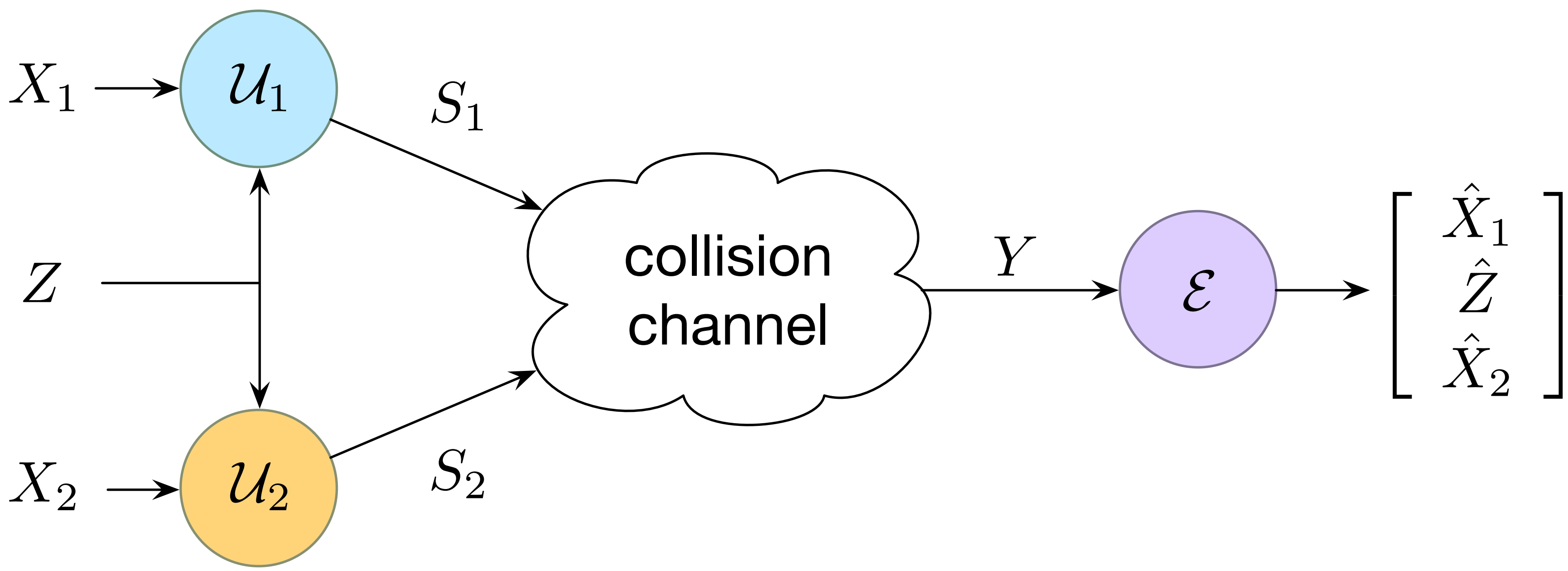
no transmission $\varnothing$

collision $\mathfrak{C}$

The collision channel is fundamentally different from the packet drop channel[1,2]

1. Sinopoli et al, "Kalman filtering with intermittent observations," *IEEE TAC* 2004.
2. Gupta et al, "Optimal LQG control across packet-dropping links," *Systems and Control Letters* 2007.

Why is this problem interesting?



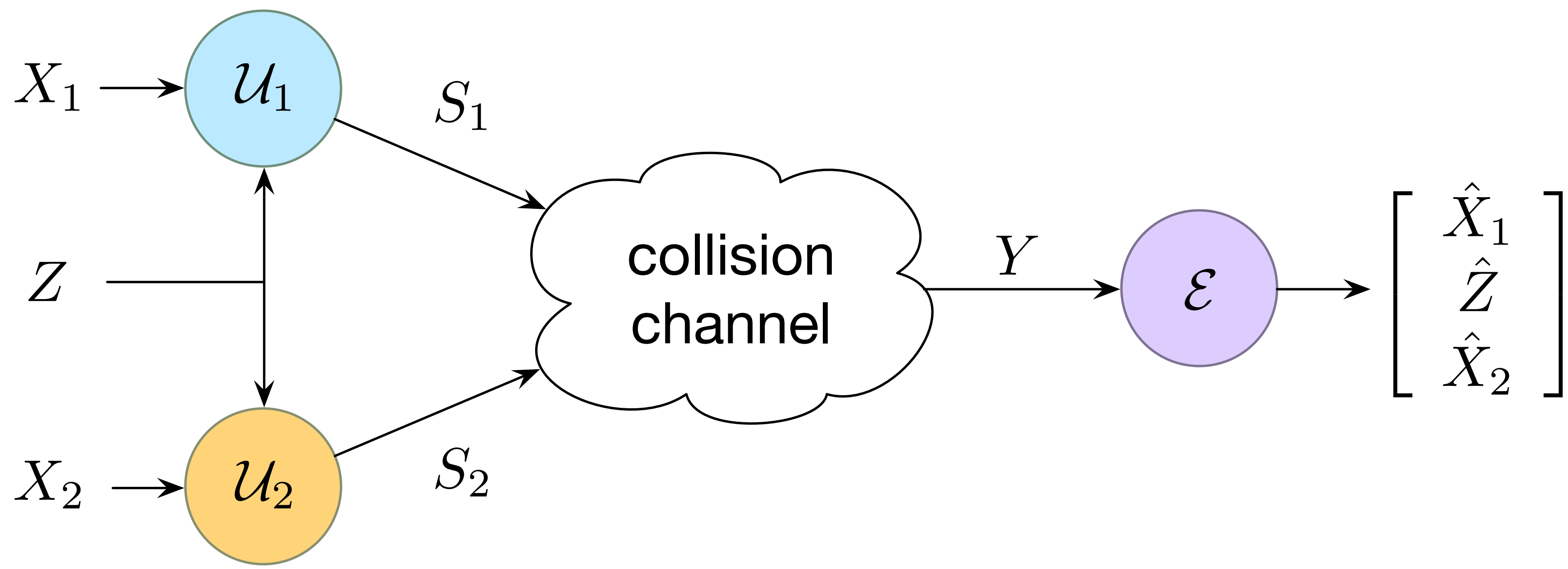
Problem

minimize $\mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (Z - \hat{Z})^2 + (X_2 - \hat{X}_2)^2\right]$

Team-decision problem $\Longrightarrow$ **Non-convex** (in most cases) **intractable**[1,2]

1. Witsenhausen, “A counterexample in optimal stochastic control,” *SIAM J. Control* 1968.
2. Tsitsiklis & Athans, “On the complexity of decentralized decision making and detection problems,” *IEEE TAC* 1985.

Why is this problem interesting?



Problem

minimize $\quad \mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (Z - \hat{Z})^2 + (X_2 - \hat{X}_2)^2\right]$

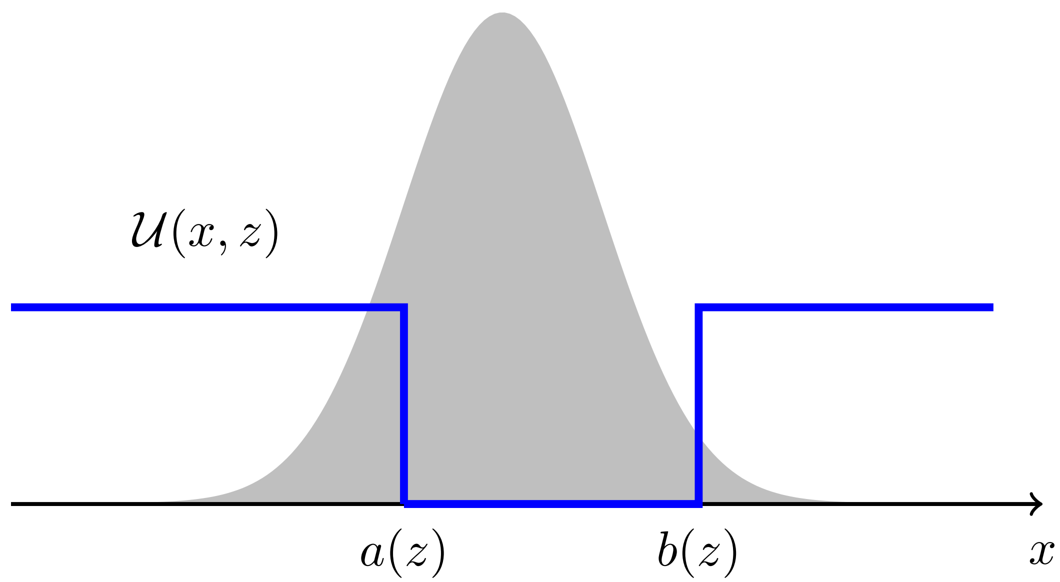
Look for a class parametrizable policies that contains an optimal strategy

1. Witsenhausen, "A counterexample in optimal stochastic control," *SIAM J. Control* 1968.
2. Tsitsiklis & Athans, "On the complexity of decentralized decision making and detection problems," *IEEE TAC* 1985.

Main result: Threshold policy on private information

Theorem:

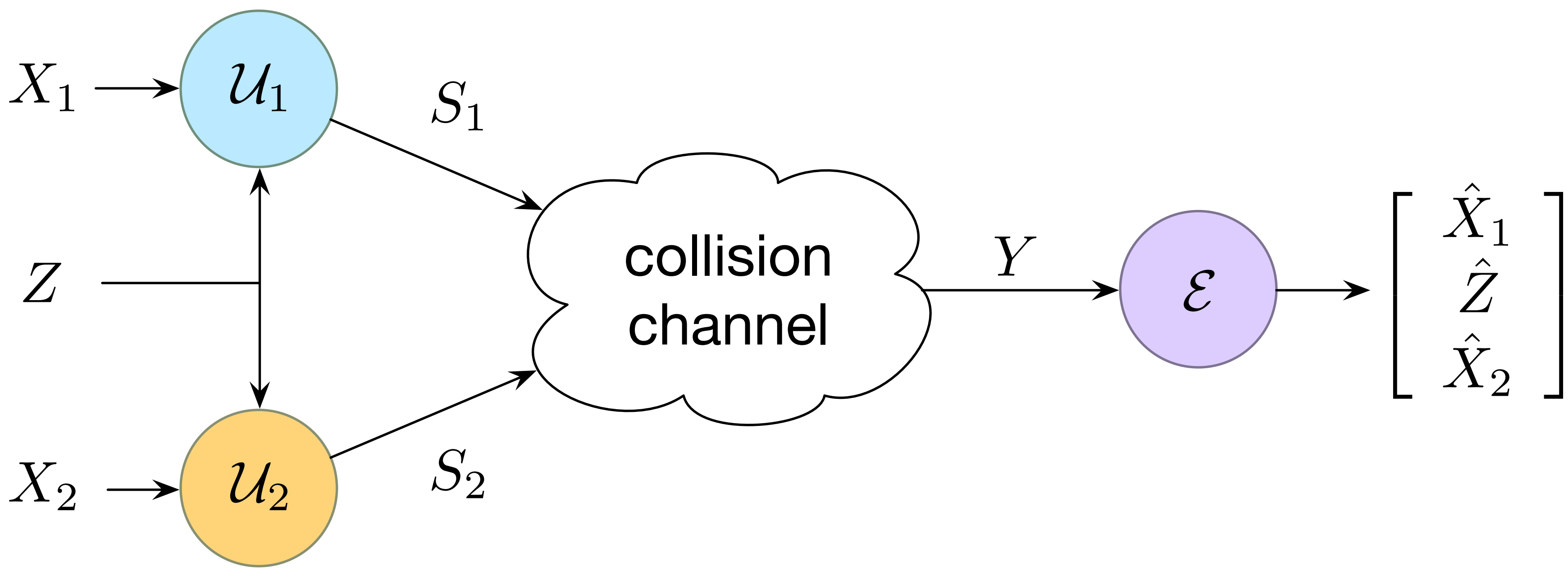
If a team-optimal pair of communication policies exist, there is a pair of **threshold policies on private information** that attains the optimal cost.



Threshold policy on private information

$$\mathcal{U}(x,z) = \begin{cases} 0 & a(z) \le x \le b(z) \\ 1 & \text{otherwise} \end{cases}$$

Step 1: Fixing the structure of the estimator



Define the class of admissible estimators $\mathbb{E}$:

$$Y = \varnothing \implies \mathcal{E}(\varnothing) = \left[\hat{x}_{1\varnothing} \ \hat{z}_{\varnothing} \ \hat{x}_{2\varnothing}\right]$$

$$Y = \mathfrak{C} \implies \mathcal{E}(\mathfrak{C}) = \left[\hat{x}_{1\mathfrak{C}} \ \hat{z}_{\mathfrak{C}} \ \hat{x}_{2\mathfrak{C}}\right]$$

$$Y = (1, z, x_1) \implies \mathcal{E}(1, z, x_1) = \left[x_1 \ z \ \hat{f}_{2\varnothing}(z)\right]$$

$$Y = (2, z, x_2) \implies \mathcal{E}(2, z, x_2) = \left[\hat{f}_{1\varnothing}(z) \ z \ x_2\right]$$

representation **points**

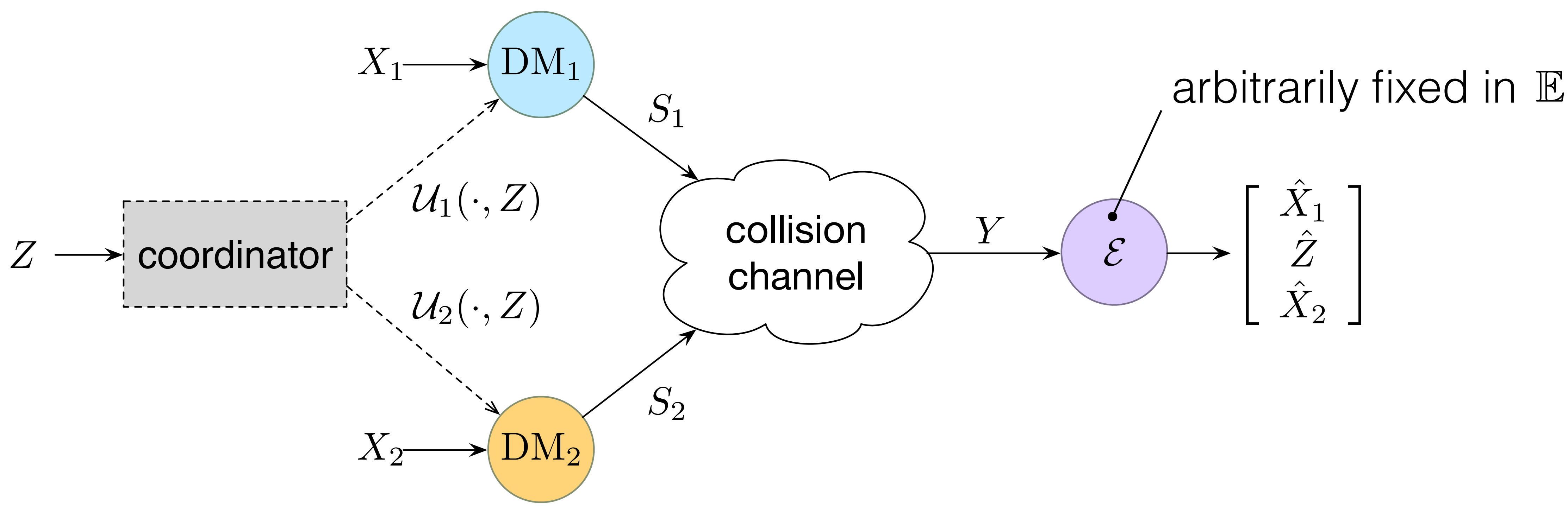
representation **functions**

$$\mathcal{E}^*(y) = \mathbf{E}\left[W \mid Y = y\right]$$

$$\mathcal{E}^* \in \mathbb{E}$$

Step 2: Common information approach

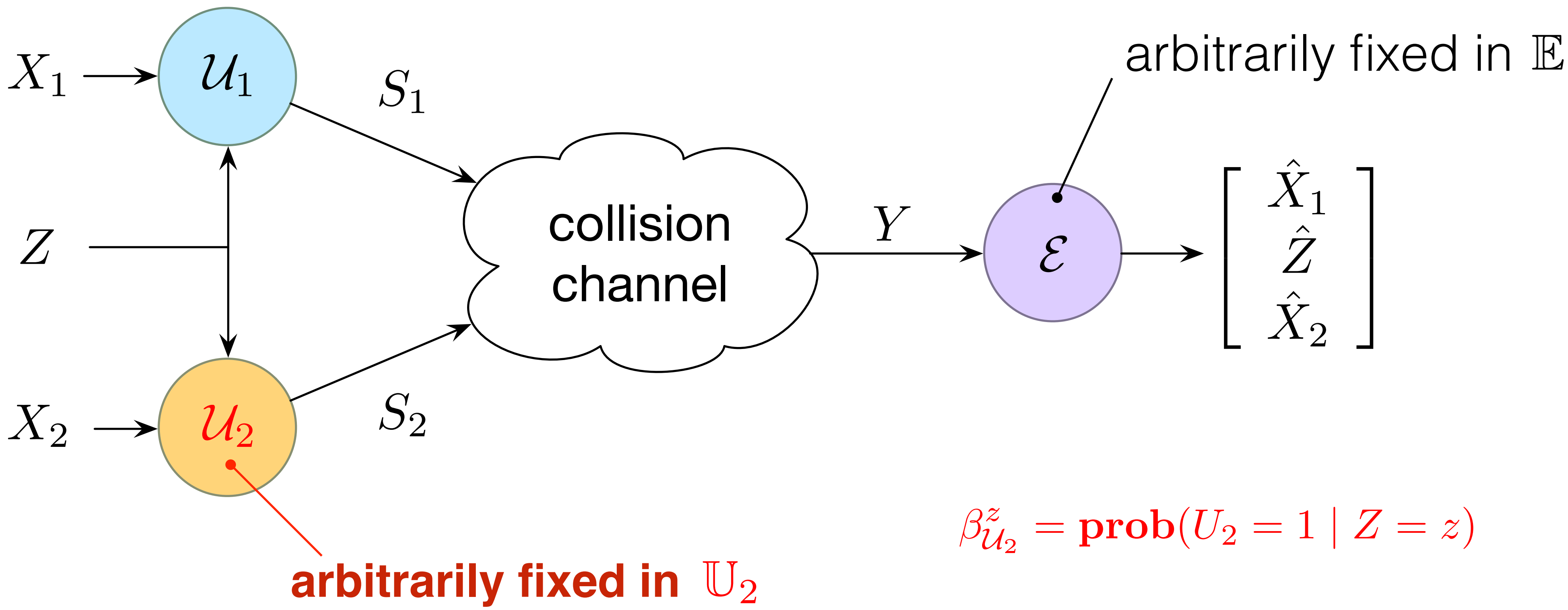
Common information[1,2] can be used to **simplify** and **characterize** optimal solutions of team problems.



$$\text{minimize} \qquad \mathcal{J}^z(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E}\left[(W - \hat{W})^T (W - \hat{W}) \mid Z = z\right]$$

1. Nayyar, Mahajan & Teneketzis, "Decentralized stochastic control with partial history sharing," *IEEE TAC* 2013.
2. Nayyar, Mahajan & Teneketzis, "The common information approach to decentralized stochastic control," Springer 2014.

Step 3: Person-by-person approach



$$\beta^z_{\mathcal{U}_2} = \mathbf{prob}(U_2 = 1 \mid Z = z)$$

$$\mathcal{J}^z(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E}\Big[(X_1 - \hat{X}_1)^2 + (Z - \hat{Z})^2 \mid Z = z\Big] + \rho^z_{\mathcal{U}_2} \mathbf{prob}(U_1 = 1 \mid Z = z) + \theta^z_{\mathcal{U}_2}$$

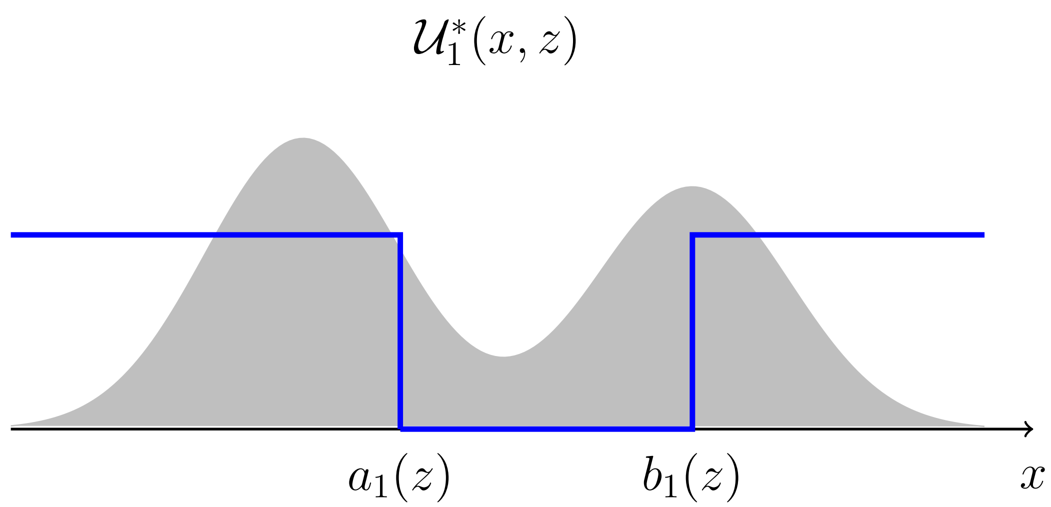
$$\begin{aligned} &\text{minimize} \quad \mathcal{J}^z(\mathcal{U}_1, \mathcal{U}_2) \\ &\text{subject to} \quad 0 \leq \mathcal{U}_1(x,z) \leq 1 \quad x \in \mathbb{X}_1 \end{aligned}$$

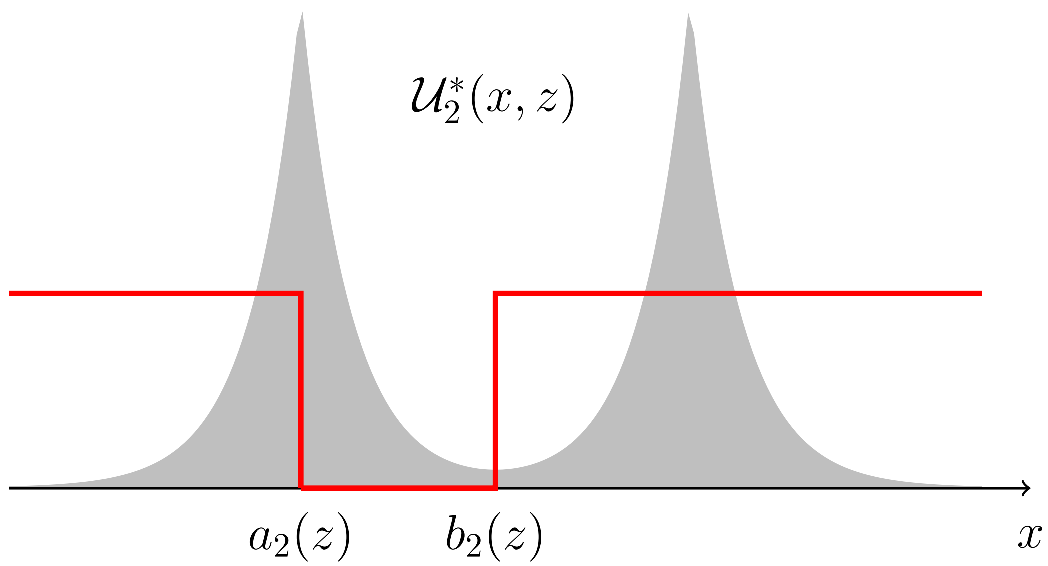
$$\mathcal{U}_1^*(x,z) = \begin{cases} 0 & \text{if } \ a_1(z) \leq x \leq b_1(z) \\ 1 & \text{otherwise} \end{cases}$$

$$a_1(x), b_1(x) = \texttt{roots}\Big\{(1 - \beta^z_{\mathcal{U}_2})\Big[(x - \hat{x}_{1\varnothing})^2 + (z - \hat{z}_\varnothing)^2\Big] + \beta^z_{\mathcal{U}_2}(x - \hat{f}_{1\varnothing}(z))^2 \Big[\beta^z_{\mathcal{U}_2}(x - \hat{x}_{1\mathfrak{c}})^2 + (z - \hat{z}_{\mathfrak{c}})^2 + \rho^z_{\mathcal{U}_2}\Big]\Big\}$$

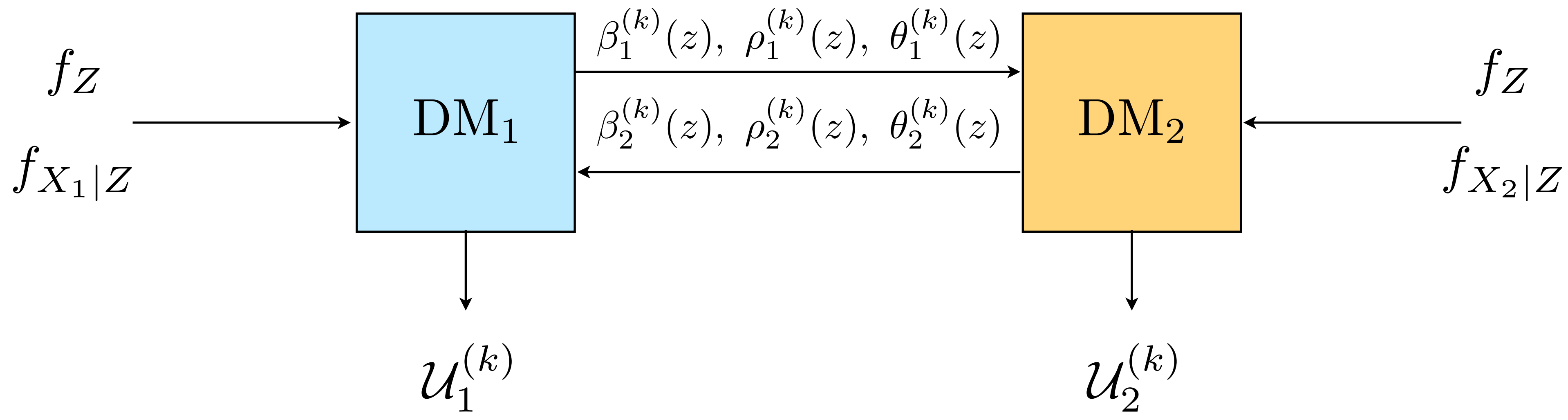
Remarks

1. Result is independent of the form of the distributions (continuity, symmetry, modality, etc…)



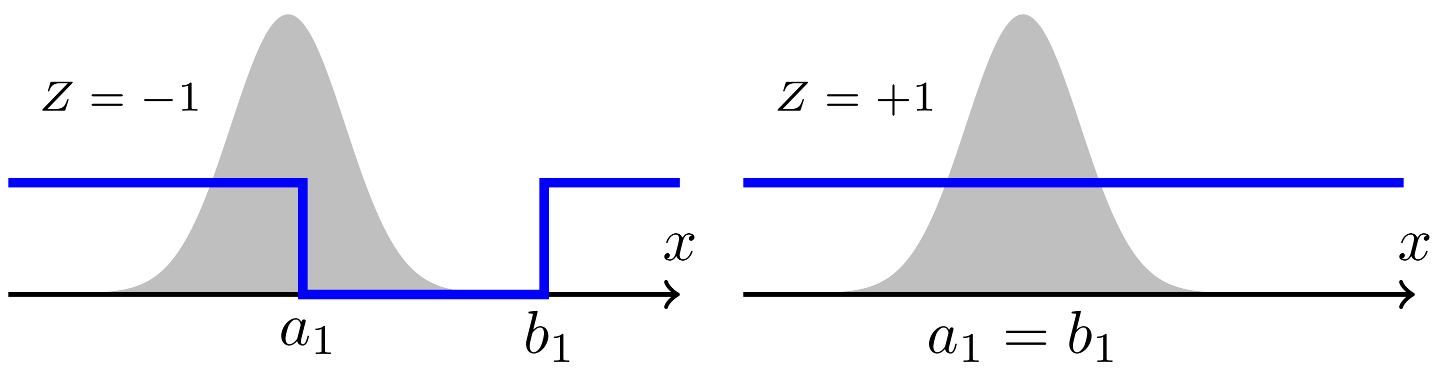


2. Alternating optimization procedure to find person-by-person optimal solutions (see paper)

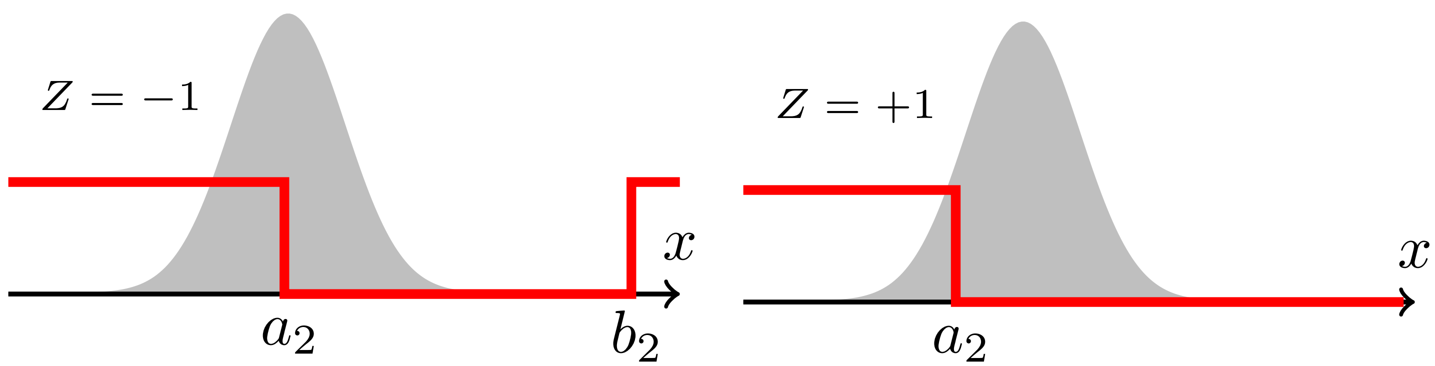


Numerical results

Example $X_1, X_2 \sim \mathcal{N}(0,1)$ $Z = \begin{cases} +1 & \text{w.p. } p \\ -1 & \text{w.p. } 1-p \end{cases}$



(a) Communication policy $\mathcal{U}_1$



(b) Communication policy $\mathcal{U}_2$

p	$\mathcal{J}^*$
0	0.54
0.1	0.59
0.2	0.63
0.3	0.68
0.4	0.73
0.5	0.78

Combination of **scheduling** and **event-based** policies.

$$p = 0.5 \implies \mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2^*) = 0.78$$

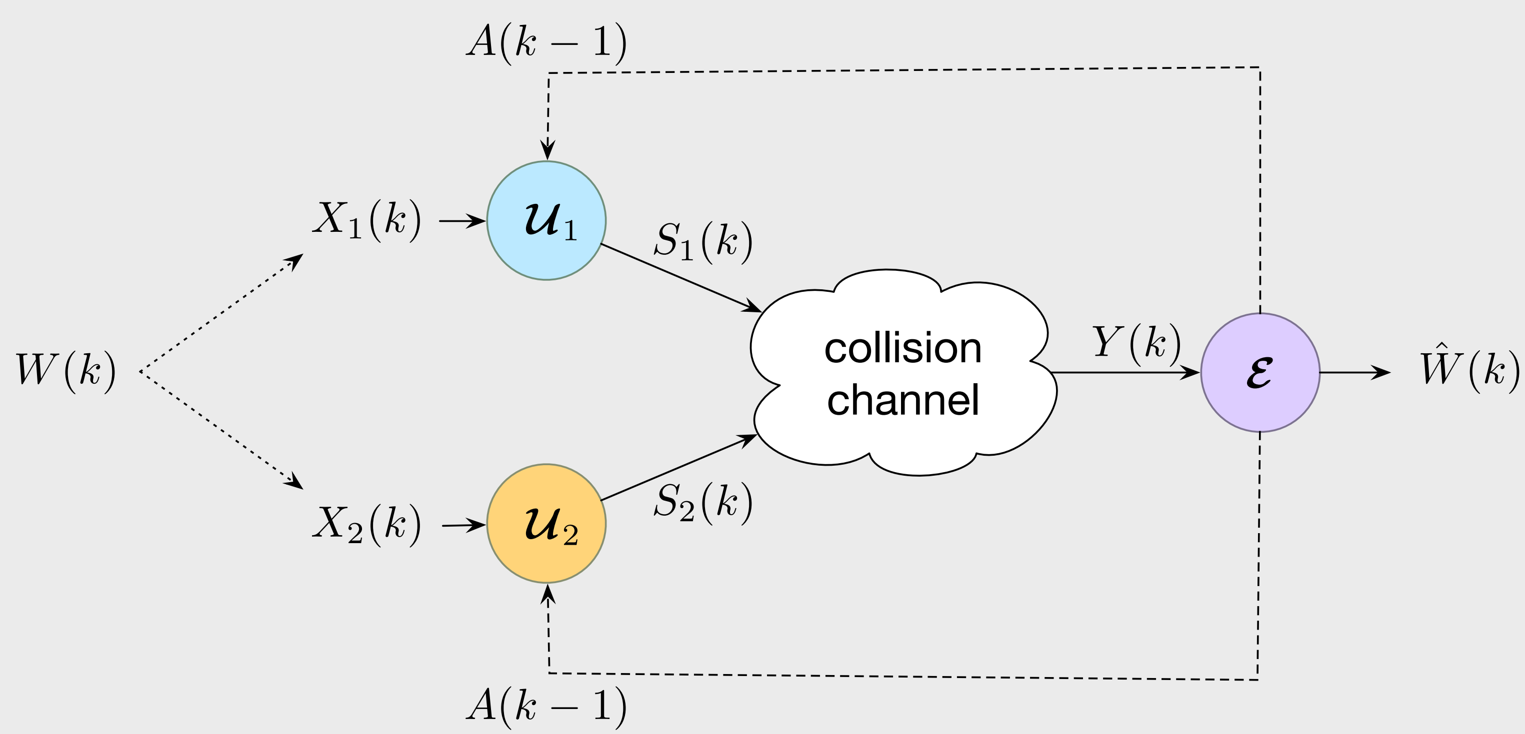
Gain of 22% over open-loop scheduling policies

Conclusion

1. Estimation over the collision channel with **dependent observations**
2. Used the **common information approach** to obtain structural results
3. **Numerical algorithm** to obtain suboptimal policies when Z **is discrete**

Future work

1. Solve the optimization problem when Z **is continuous**?
2. **Arbitrary correlation models**
3. **Sequential estimation** case with feedback (acknowledgments)



Appendix

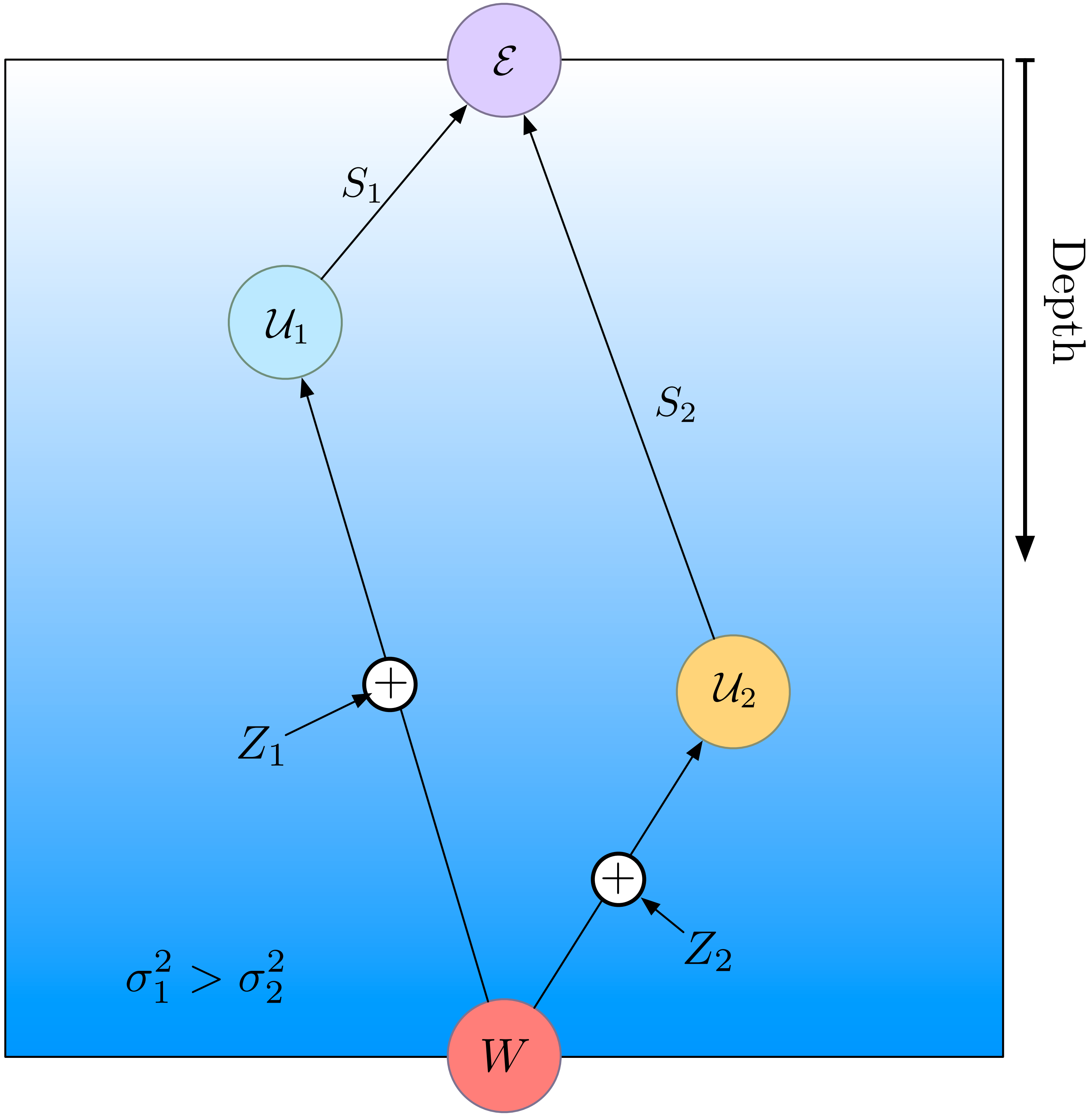
Mobile sensors: the capture effect[1]

Spatial unfairness[2]

In a **collision**, the packet transmitted by the node **closest to the fusion center survives** and the **others are lost**.

Collision aware sensor placement problem:

Choose the location that optimizes the performance of the system subject to packet collisions



1. Leentvaar and Flint, "The Capture Effect in FM Receivers," *IEEE TComm* 1976.
2. Syed et al., "Comparison and Evaluation of the T-Lohi MAC for UASN," *IEEE JSAC* 2008.