Optimal sensor scheduling strategies for networked estimation

Marcos Vasconcelos

mvasconc@usc.edu

www.terpconnect.umd.edu/~marcos

Ashutosh Nayyar **Urbashi Mitra**

Dept. of Electrical Engineering

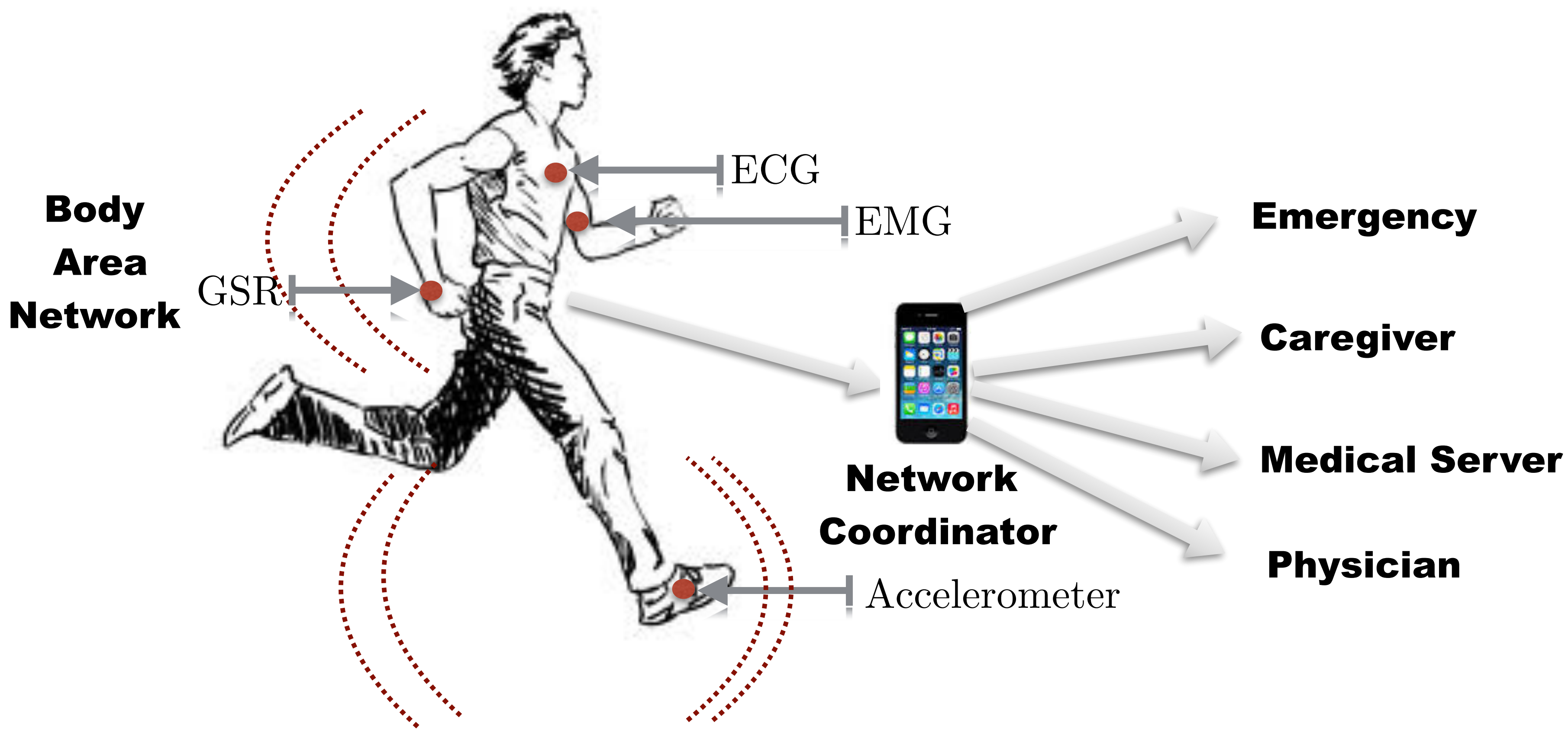
University of Southern California

56th IEEE Conference on Decision and Control

12-15 December 2017 - Melbourne, Australia

Body area networks

System coupling bio-sensors on people and wireless networks



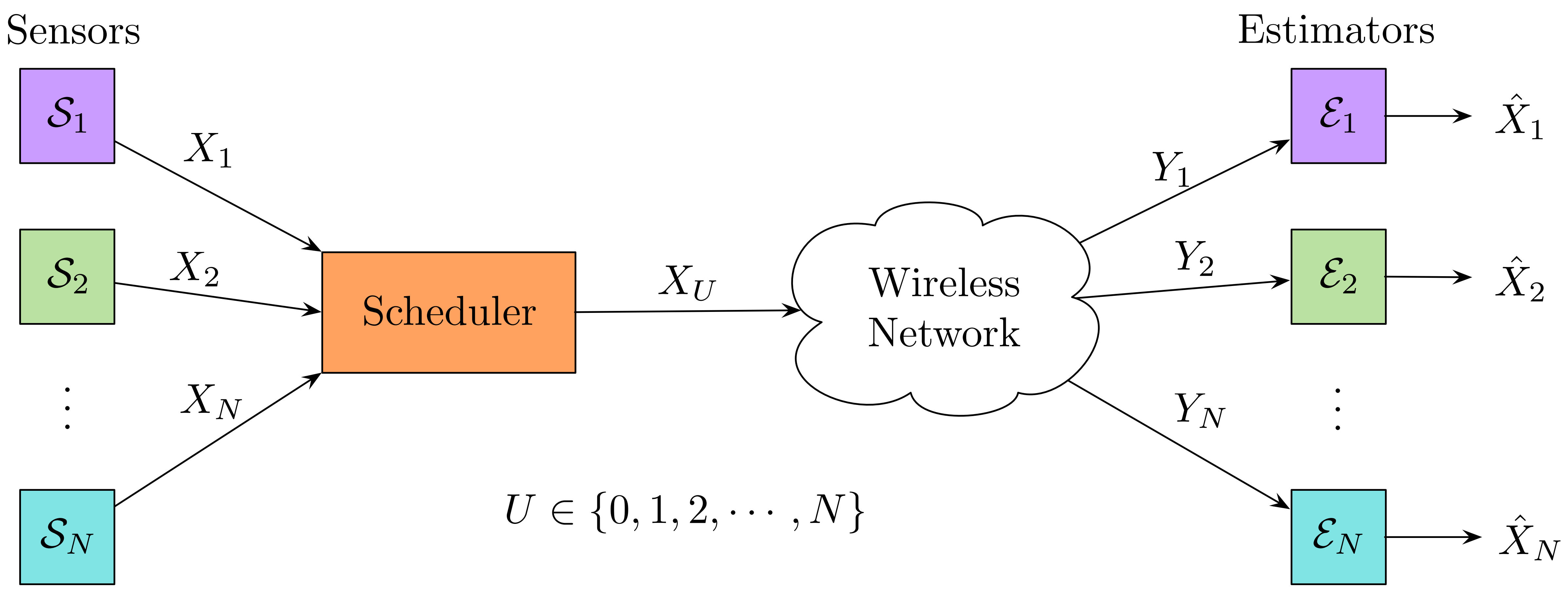
Goals

1. Real-time monitoring
2. Feedback and interventions

Design challenges

1. Data heterogeneity
2. Communication constraints
3. Energy constraints

Basic framework

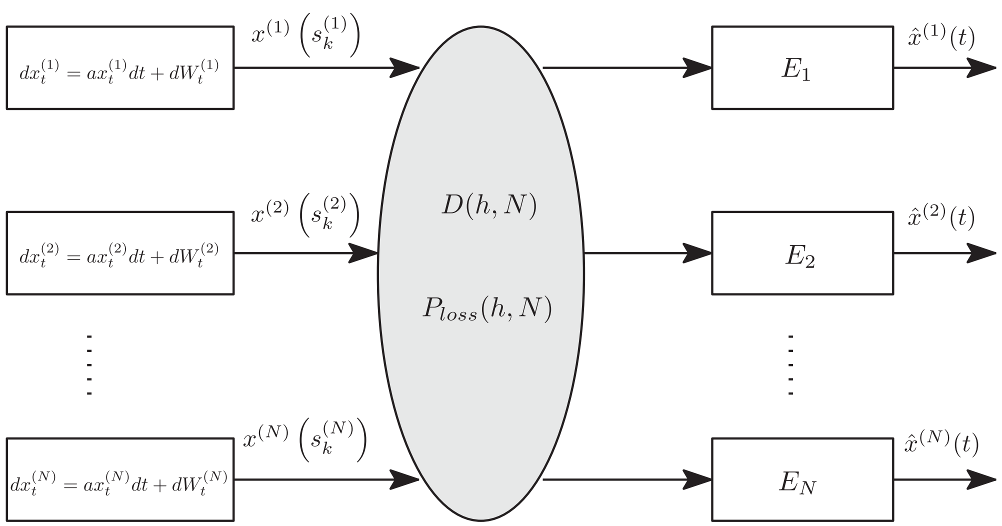


Communication constraint

At most one packet can be transmitted

Related work: Estimation

Rabi et al - *Int. J. Rob. Non. Cont.* 2009

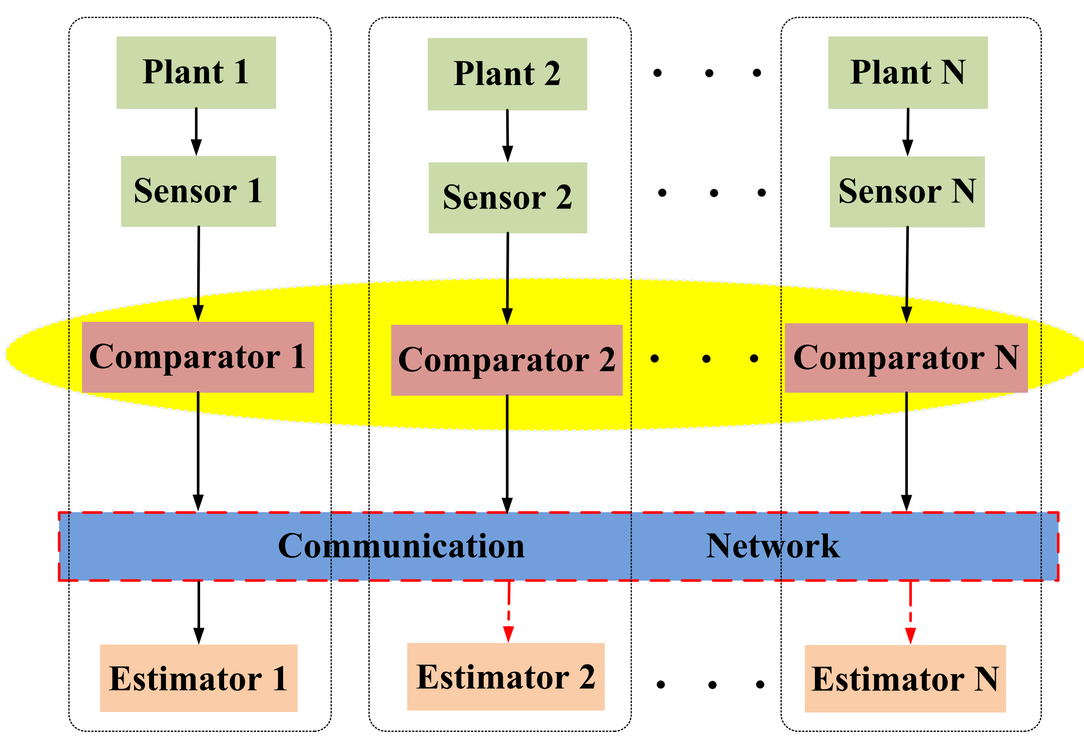


Shared channel, contention-based MAC

- CSMA scheduling

$$J_e \triangleq \frac{1}{N}\sum_{i=1}^{N} \limsup_{M\to\infty} \frac{1}{M}\int_0^M \mathbb{E}[(x_t^{(i)} - \hat{x}_t^{(i)})^2]\,dt$$

Xia, Gupta and Antsaklis - *TAC 2017*

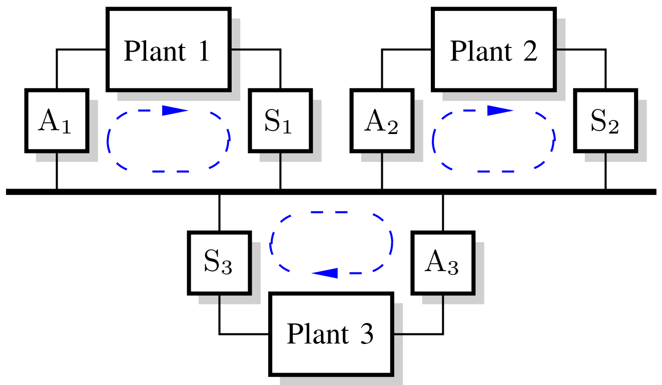


- Scheduling fixed
  - Static: TDMA, randomized, …
  - Dynamic: **max-scheduling**

$$J \triangleq \sum_{i=1}^{N} \lim_{t\to\infty} \frac{1}{t}\sum_{k=0}^{t} \mathbb{E}\left\{e_i^{\text{dec}}(k)\,[e_i^{\text{dec}}(k)]^\top\right\}$$

Related work: Control

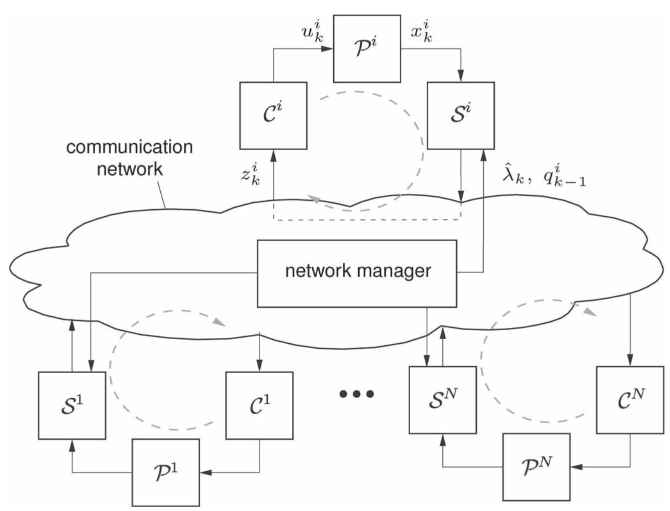
Cervin and Henningsson - *CDC* 2008



- Scheduling fixed: TDMA, FDMA or CSMA

$$J_i = \lim_{t\to\infty} \frac{1}{t} \mathrm{E} \int_0^t (x_i(s))^2 ds$$

Molin and Hirche, *TAC* 2014

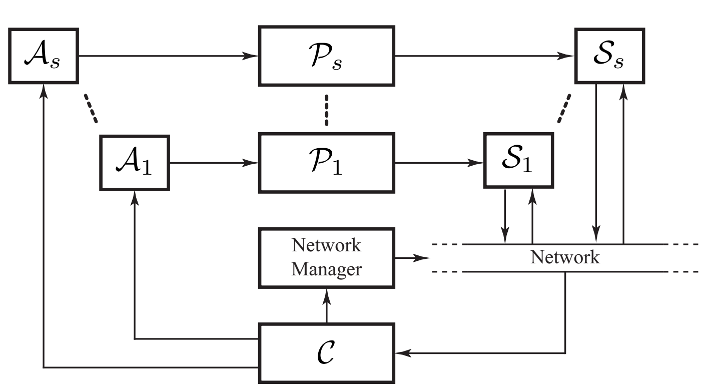


- Joint CSMA type scheduling and control

$$\min_{\substack{\gamma^1,\dots,\gamma^N\\ \pi^1,\dots,\pi^N}} \sum_{i=1}^{N} J^i$$

$$J^i = \limsup_{T\to\infty} \frac{1}{T} \mathsf{E} \left[\sum_{k=0}^{T-1} \left(x_k^i\right)^{\mathrm{T}} Q_x^i x_k^i + \left(u_k^i\right)^{\mathrm{T}} Q_u^i u_k^i \right]$$

Henriksson et al, *TCST* 2015



- Model predictive control and scheduling

$$\sum_{l=0}^{\infty} \left(\|x_\ell(k_\ell + l)\|_{Q_\ell}^2 + \|u_\ell(k_\ell + l)\|_{R_\ell}^2 \right)$$

Our goal

No assumptions on the scheduler or the estimators

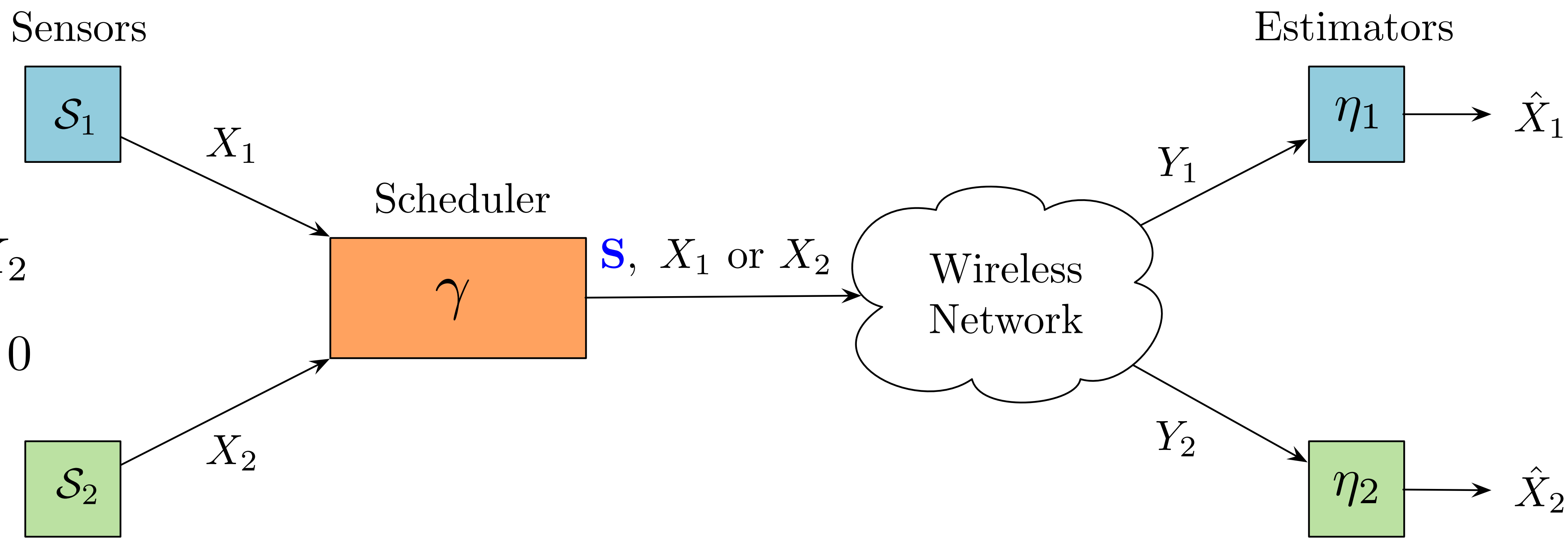
Find jointly optimal scheduling and estimation policies

One-shot problem

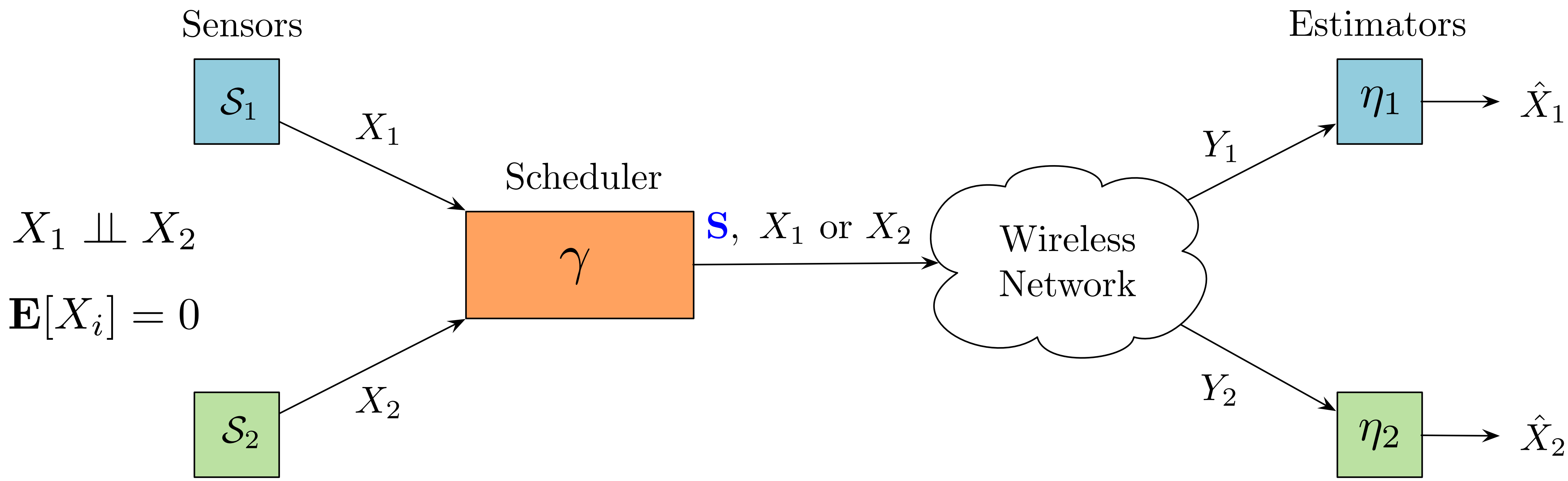
Simplest problem: two sensors

$$X_1 \perp\!\!\!\perp X_2$$

$$\mathbf{E}[X_i] = 0$$

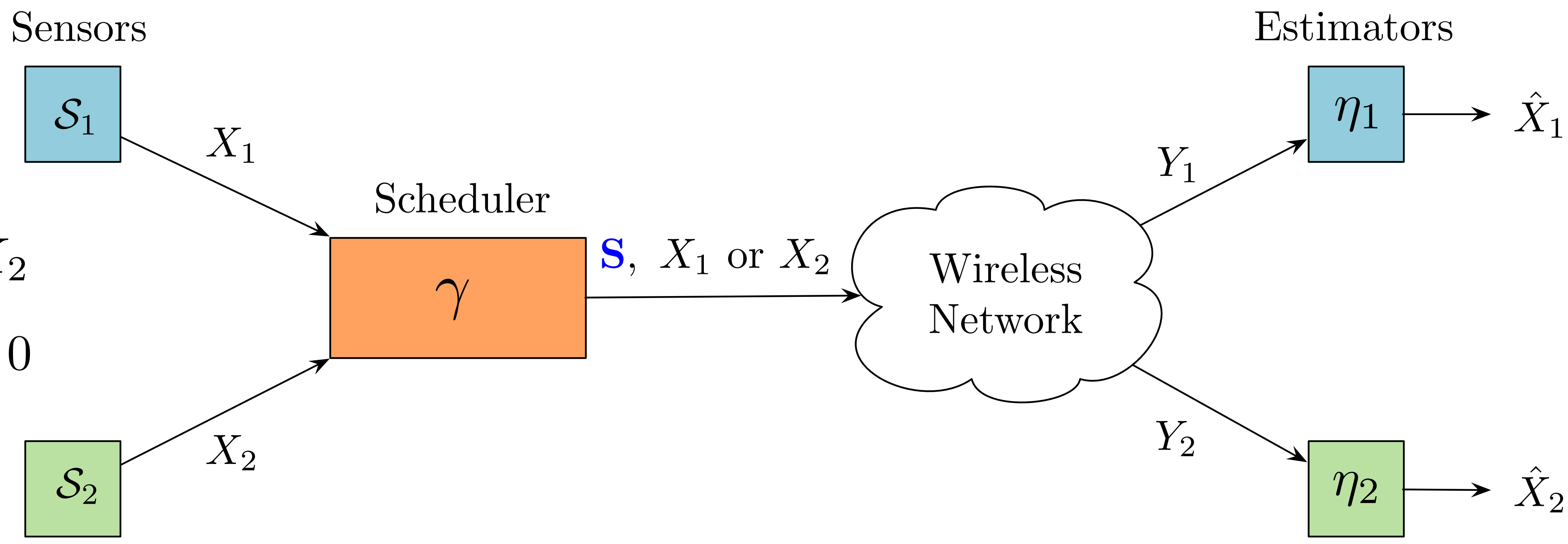


Simplest problem: two sensors



Decision variable	**Scheduling policy**	**Estimation policy**
$U \in \{0, 1, 2\}$	$U = \gamma(X_1, X_2)$	$\hat{X}_i = \eta_i(Y_i)$

Simplest problem: two sensors



$X_1 \perp\!\!\!\perp X_2$

$\mathbf{E}[X_i] = 0$

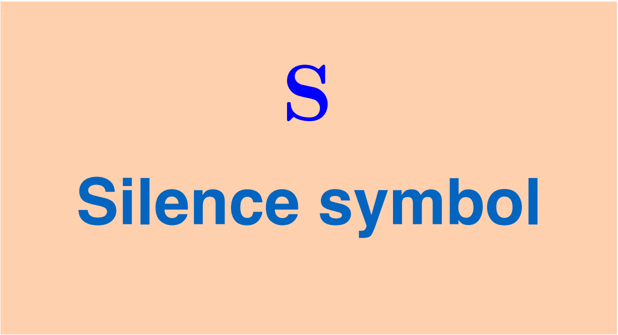
Decision variable	**Scheduling policy**	**Estimation policy**
$U \in \{0, 1, 2\}$	$U = \gamma(X_1, X_2)$	$\hat{X}_i = \eta_i(Y_i)$

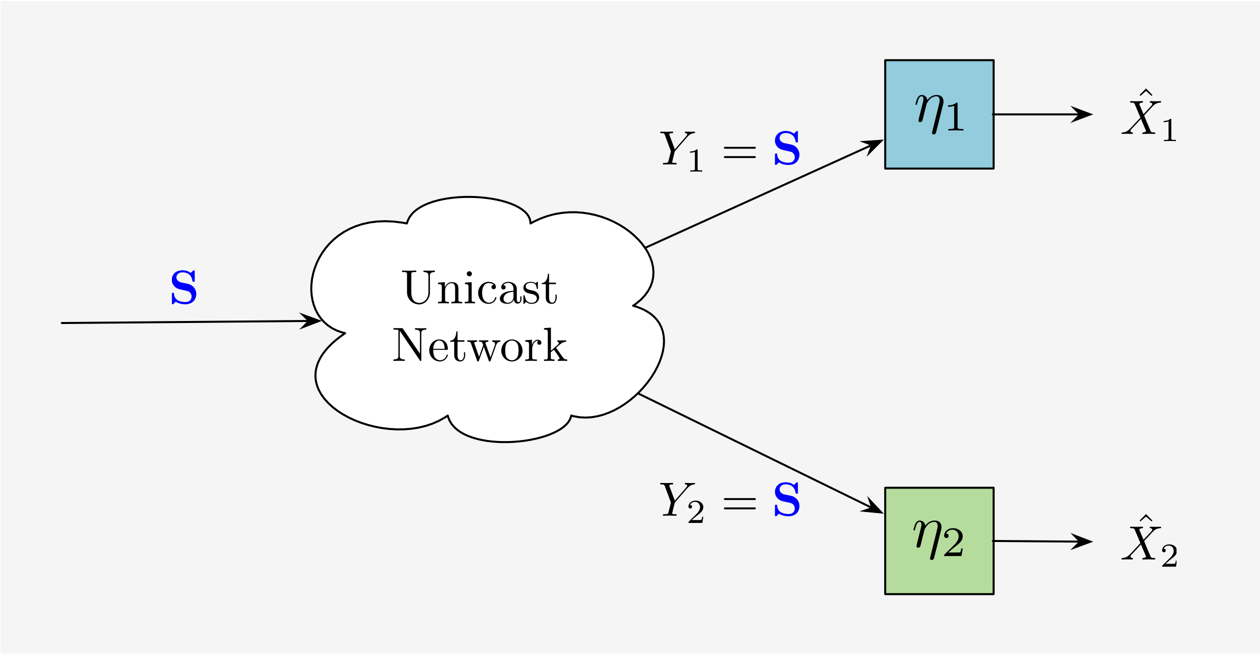
Find **scheduling** and **estimation** policies that **jointly** minimize the following cost

$$\underset{\gamma, \eta_1, \eta_2}{\text{minimize}} \quad \mathcal{J}(\gamma, \eta_1, \eta_2) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right] + c \cdot \mathbf{P}(U \neq 0)$$

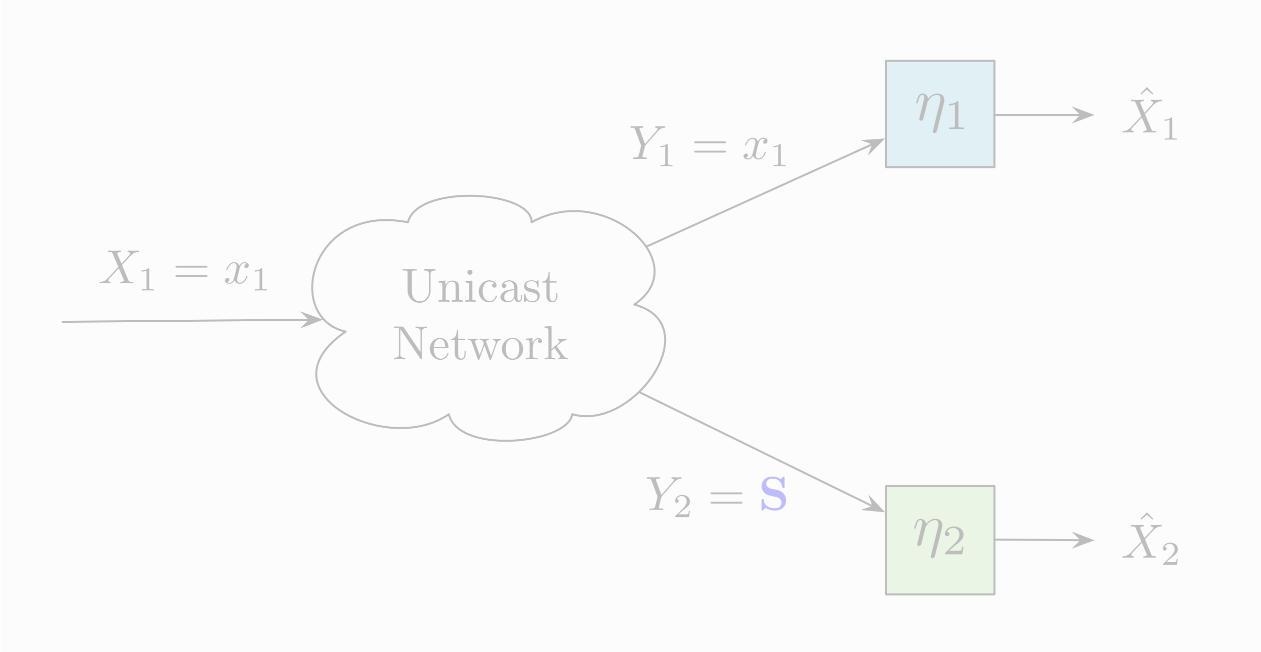
Unicast information structure

$U = 0$

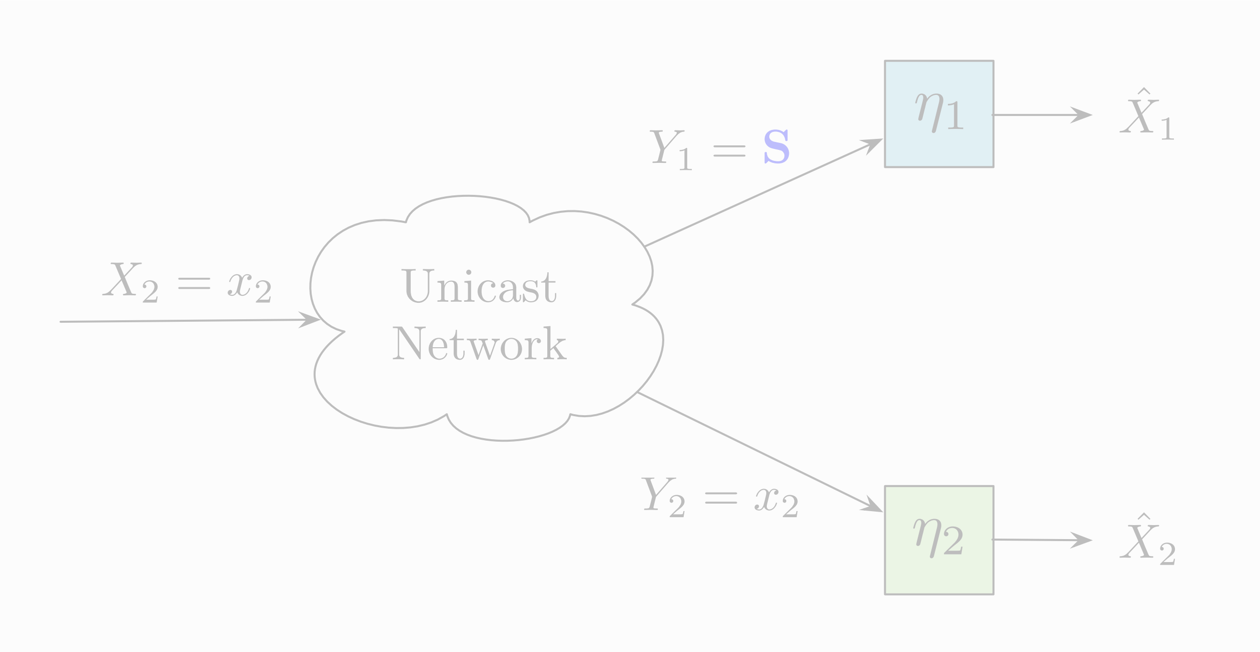




$U = 1$



$U = 2$

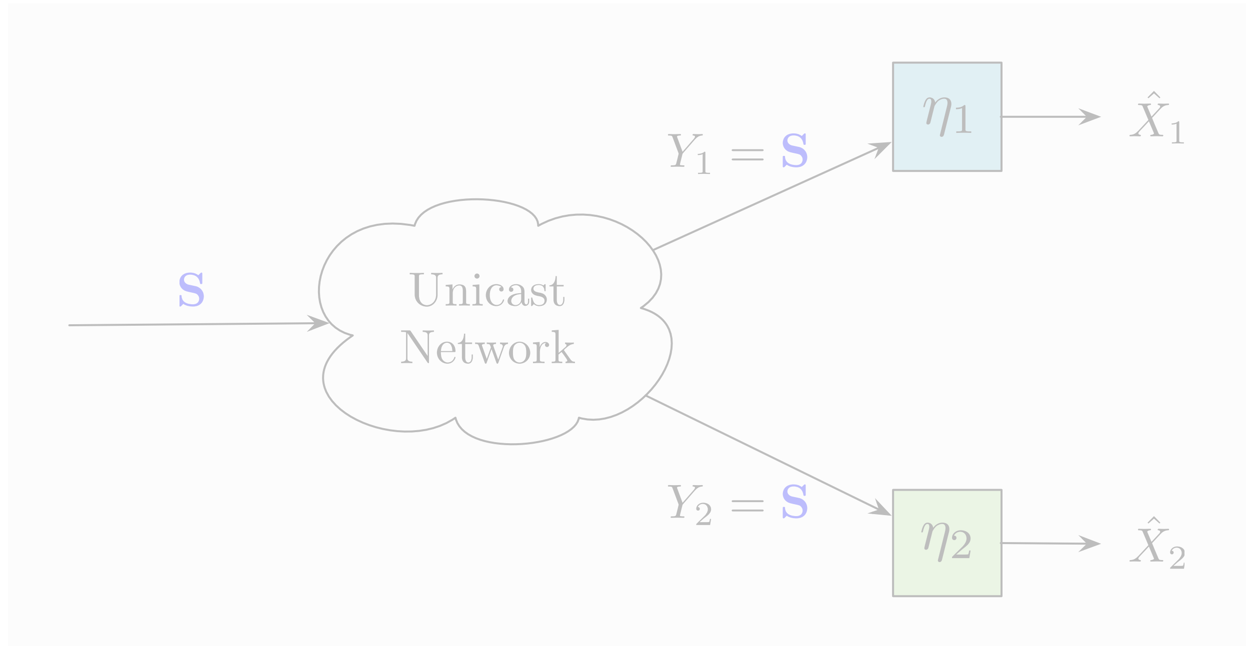


Unicast information structure

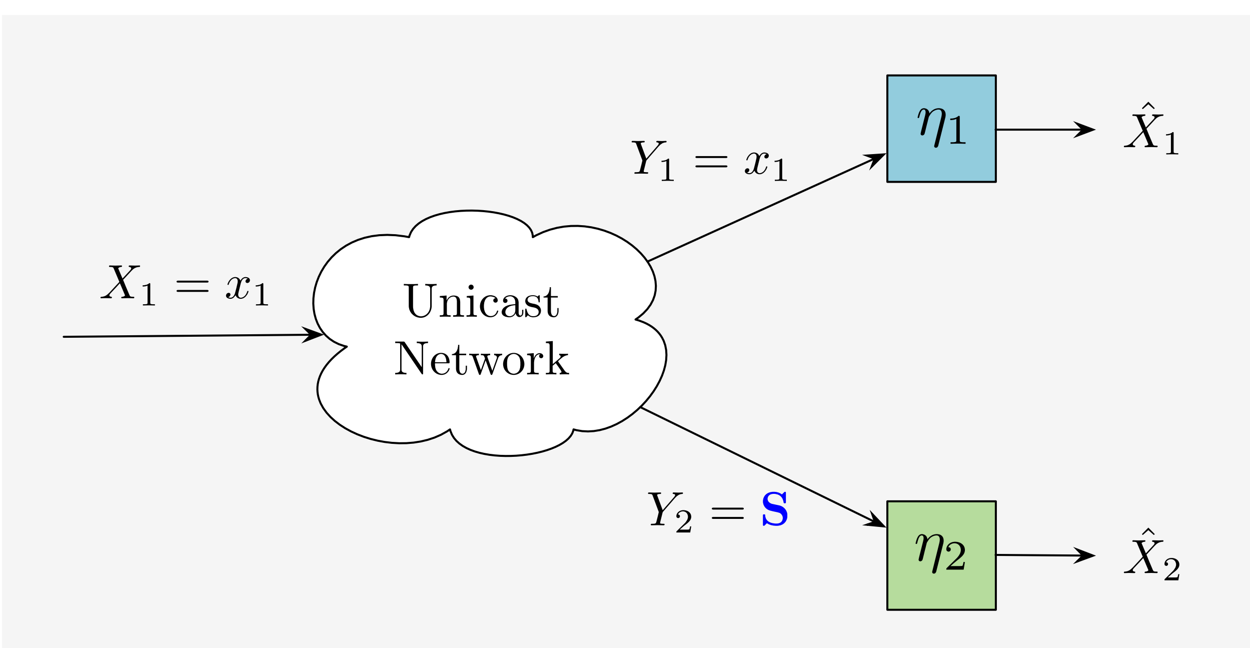
S

Silence symbol

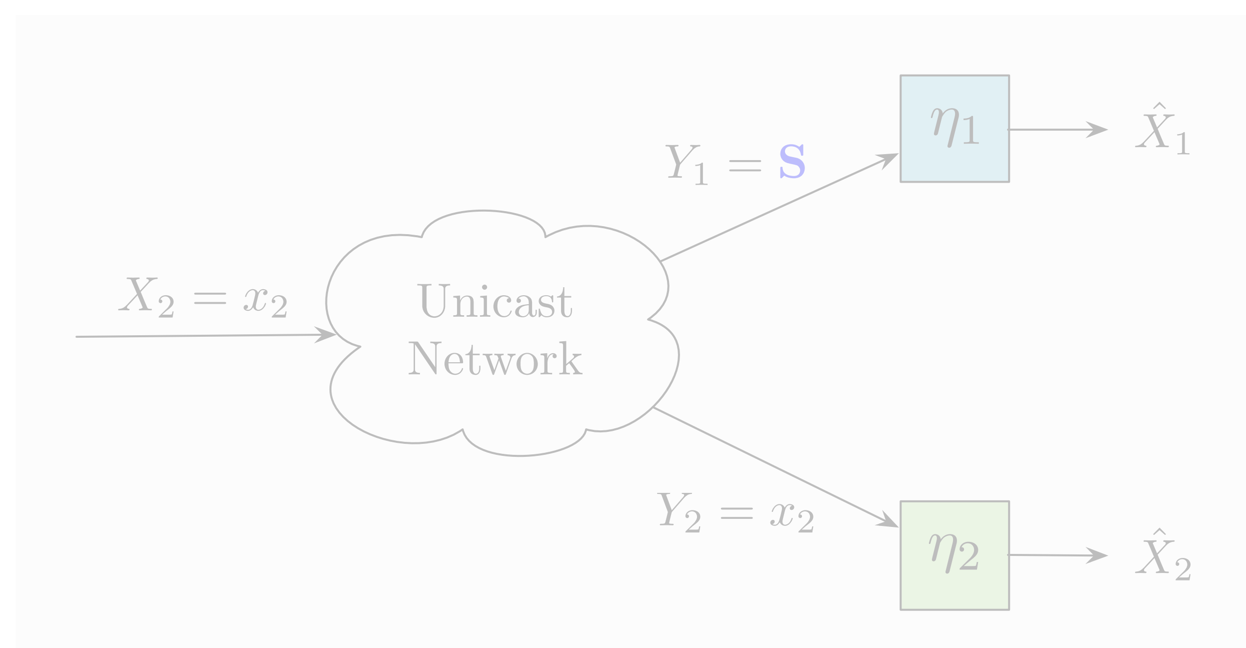
$U = 0$



$U = 1$



$U = 2$

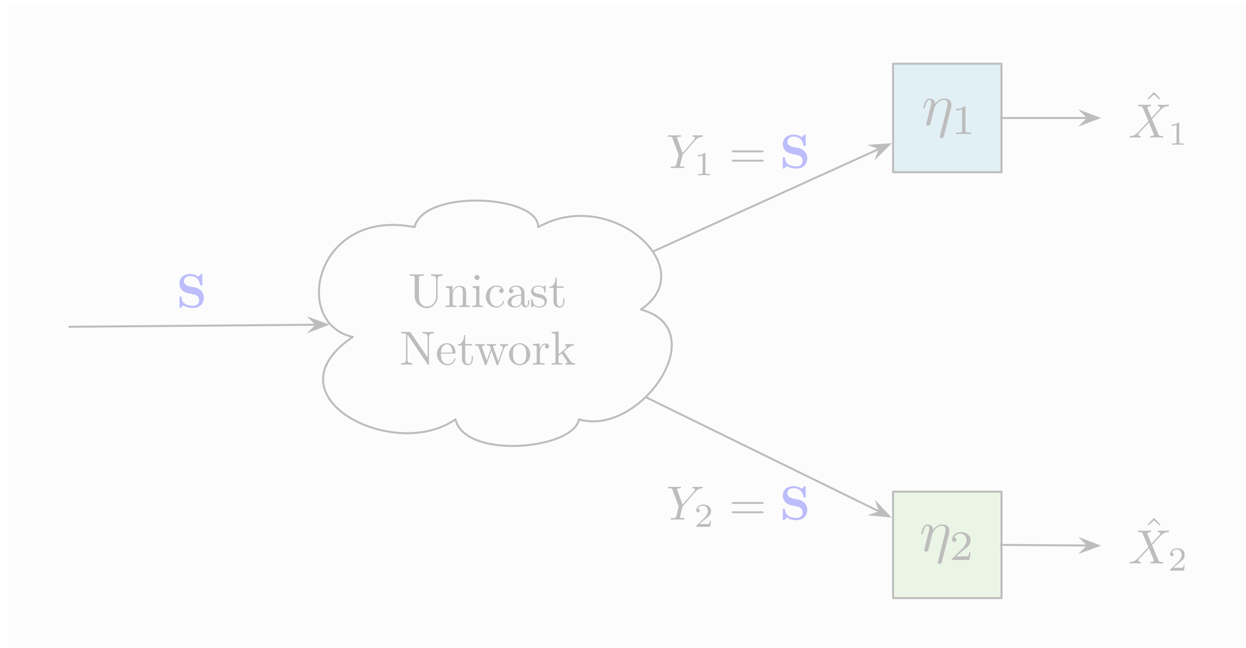


Unicast information structure

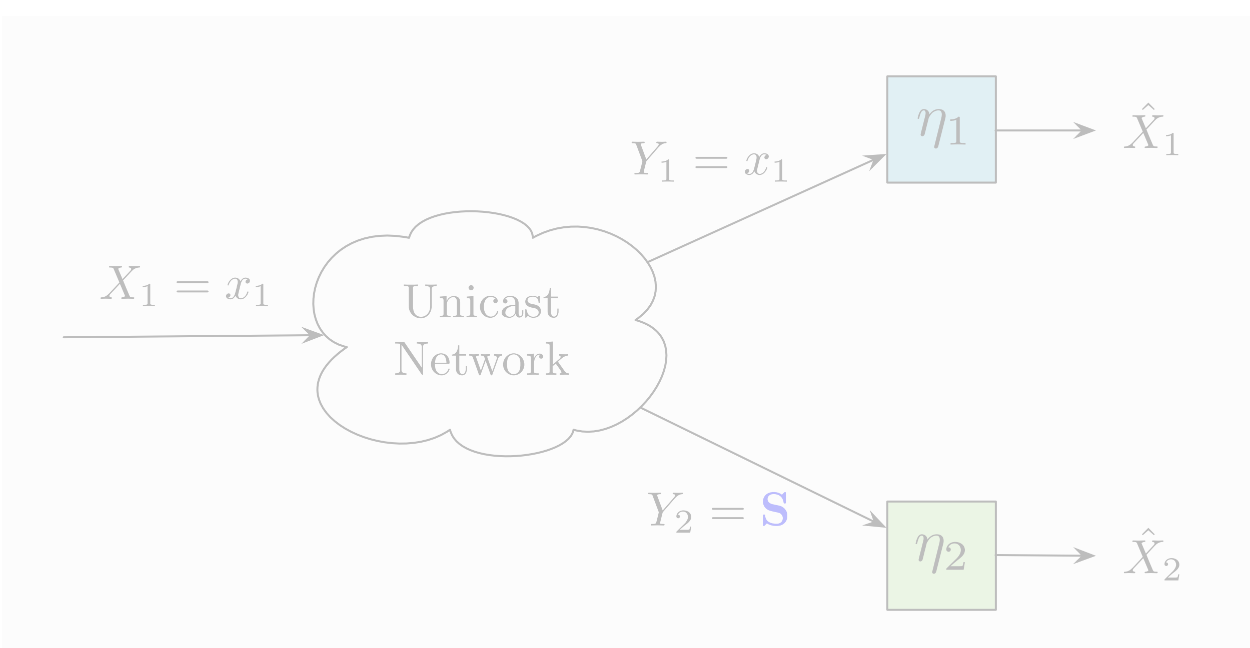
S

Silence symbol

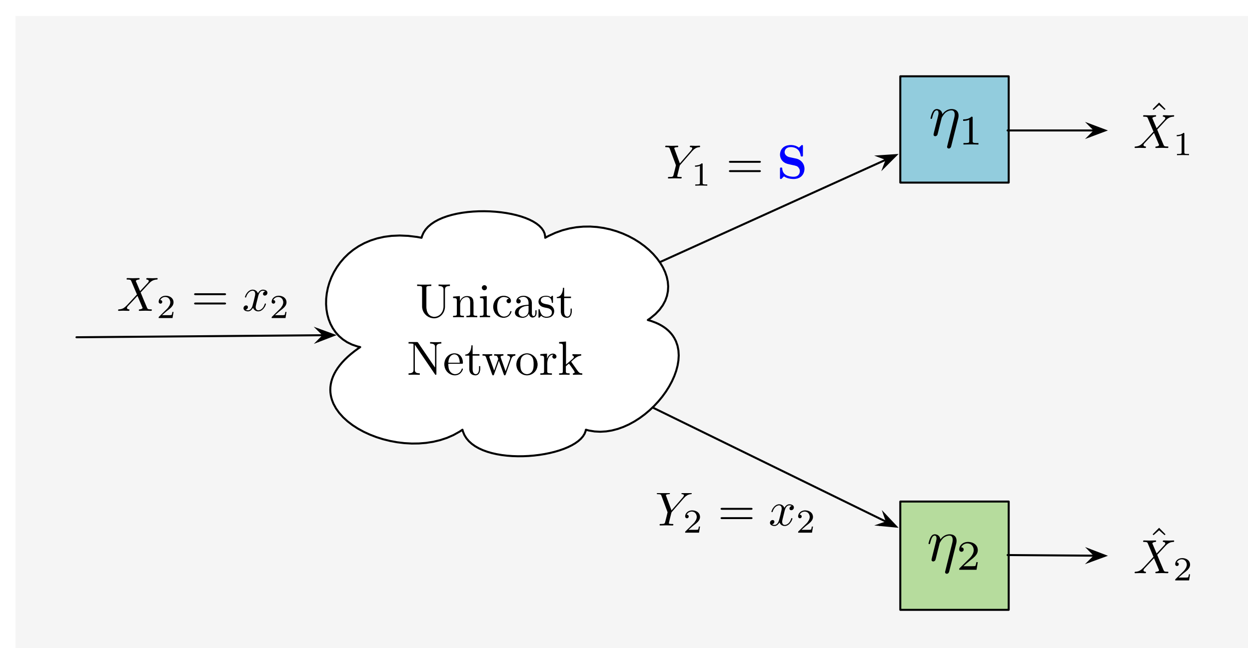
$U = 0$



$U = 1$



$U = 2$



Signaling

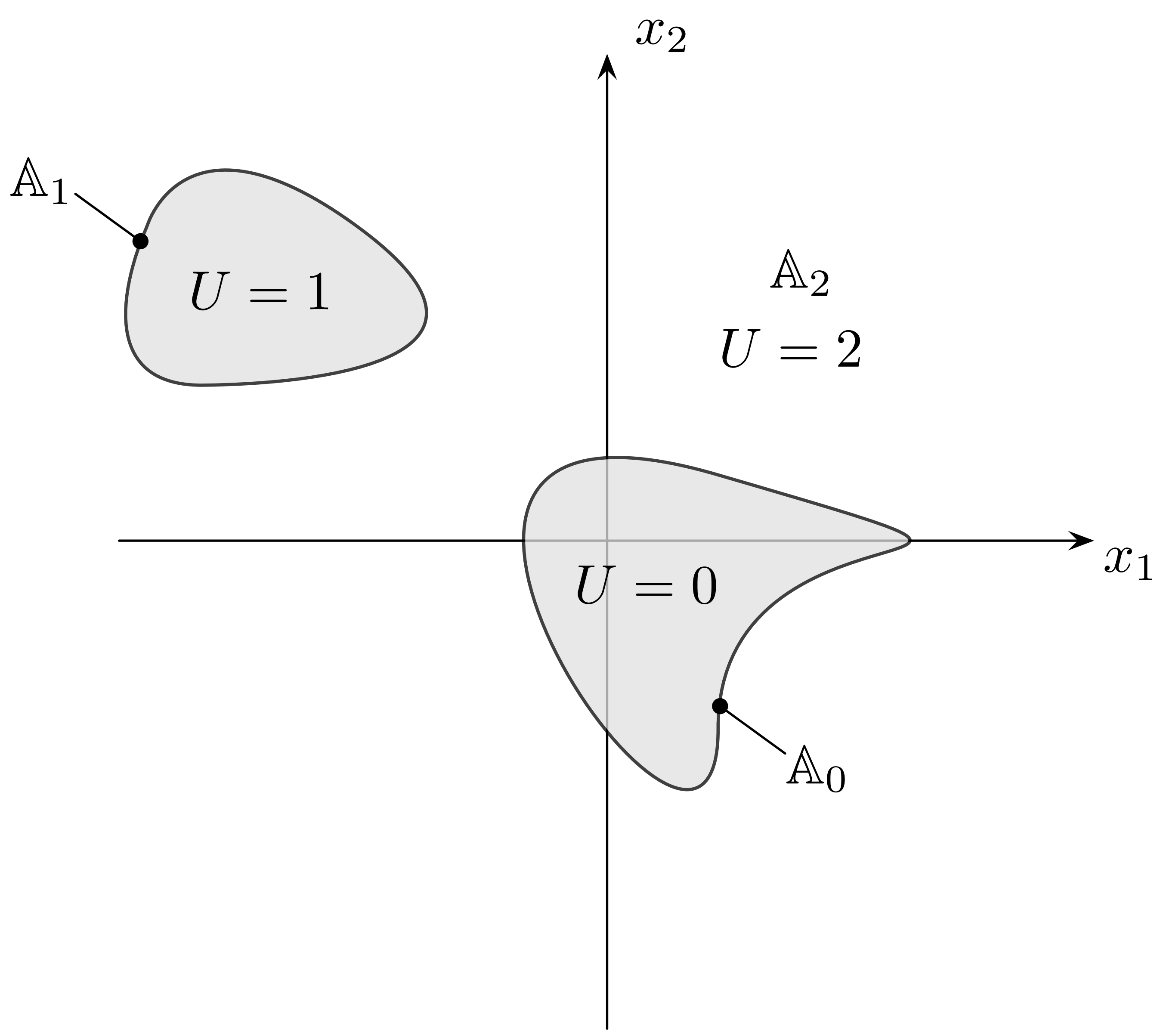
Coupling between **scheduling** and **estimation** policies

$$\underset{\gamma,\eta_1,\eta_2}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta_1,\eta_2) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right] + c \cdot \mathbf{P}(U \neq 0)$$

Nonconvex!

Signaling

$$\mathcal{J}(\gamma,\eta_1,\eta_2) = \mathbf{E}\left[(X_1-\hat{X}_1)^2 + (X_2-\hat{X}_2)^2\right] + c\cdot\mathbf{P}(U\neq 0)$$



$\hat{X}_2$ always depends on X_1

even if $X_1 \perp\!\!\!\perp X_2$

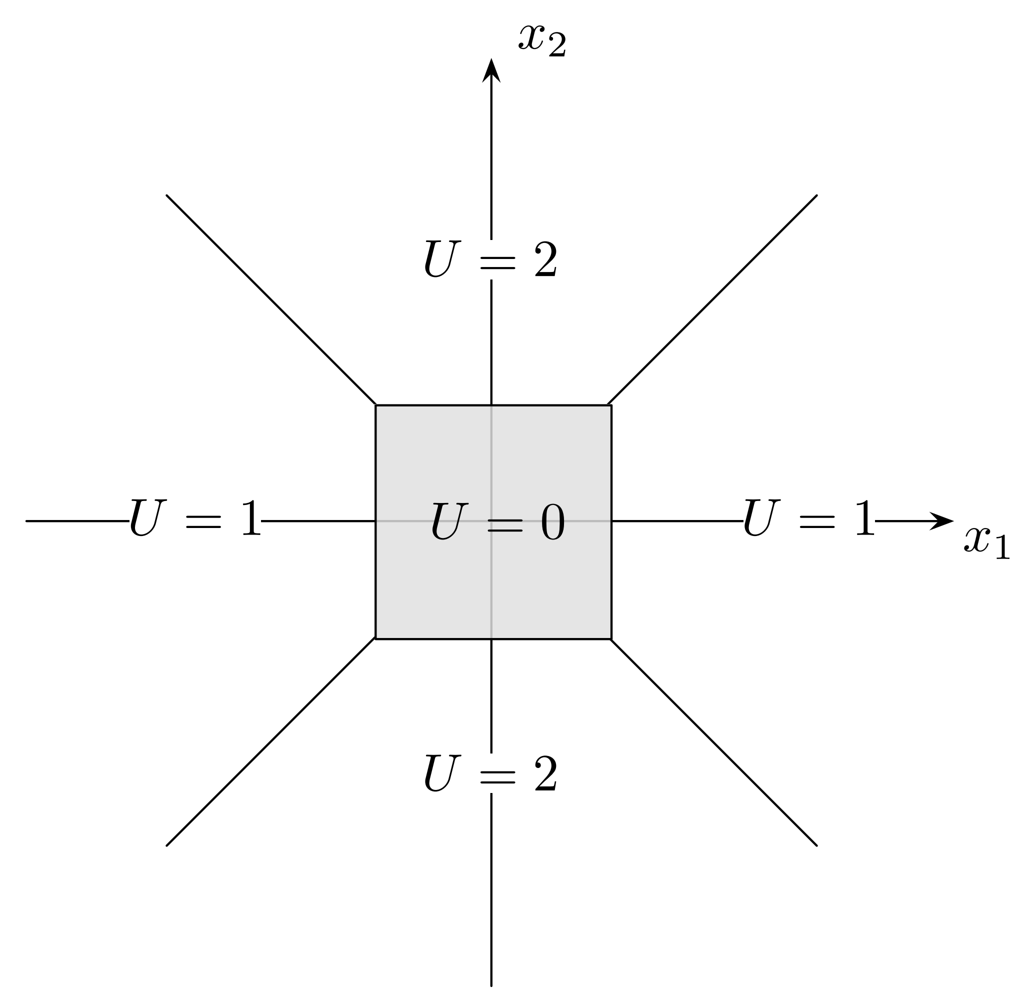
$$\begin{aligned}\hat{X}_2 &= \mathbf{E}[X_2 \mid Y_2 = \mathbf{S}] \\ &= \mathbf{E}[X_2 \mid (X_1,X_2)\in\mathbb{A}_0\cup\mathbb{A}_1]\end{aligned}$$

$\hat{X}_2 \neq \mathbf{E}[X_2]$, in general

Max-scheduling

Max-scheduling policy

$$\gamma^{\max}(x_1, x_2) \triangleq \begin{cases} 0 & \text{if } \ |x_1|, |x_2| \leq \sqrt{c} \\ \arg\max\limits_i \ |x_i| & \text{otherwise} \end{cases}$$



Estimation policy

$$\eta_i^{\text{mean}}(y) \triangleq \begin{cases} 0 & \text{if } \ y = \mathbf{S} \\ x_i & \text{if } \ y = x_i \end{cases}$$

Main result

Theorem

$$X_1 \perp\!\!\!\perp X_2$$

f_{X_1} and f_{X_2} are **continuous**, **symmetric** and **unimodal** densities

$(\gamma^{\max}, \eta_1^{\text{mean}}, \eta_2^{\text{mean}})$ is a **globally optimal solution**

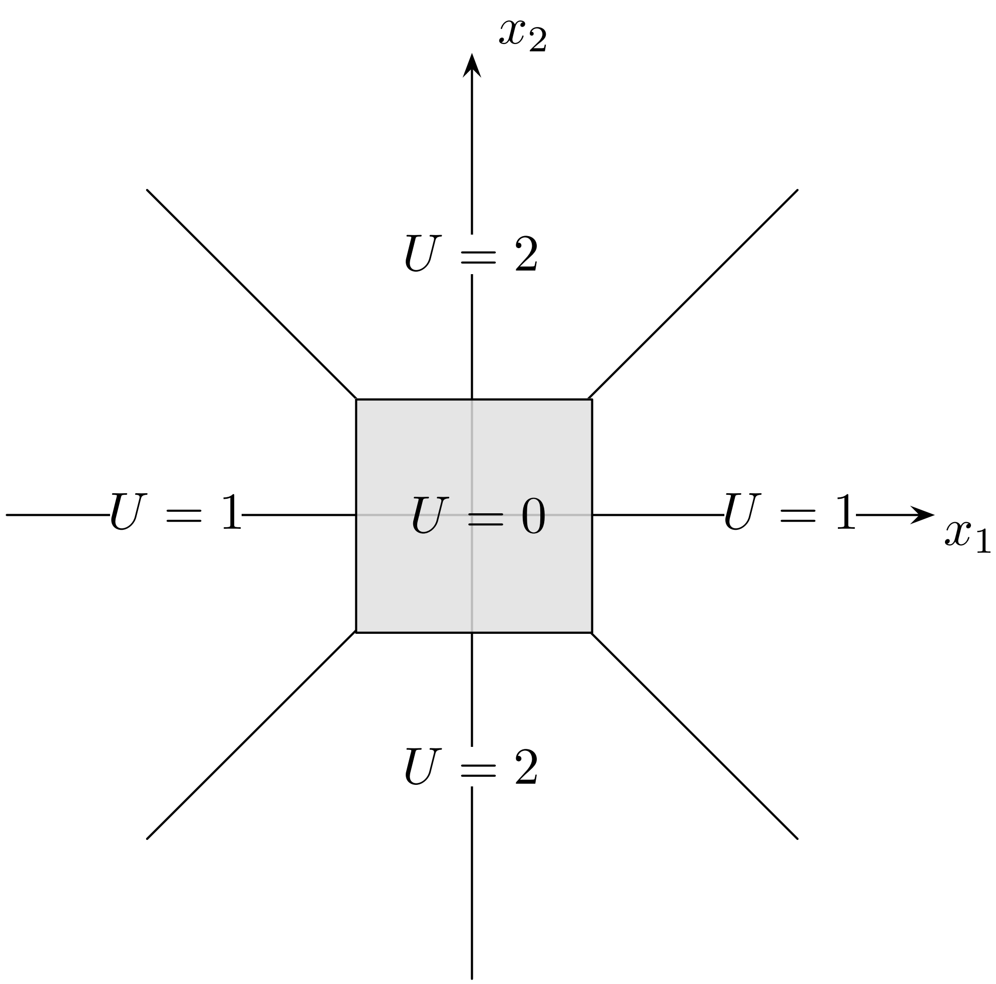
Main result

Theorem

$$X_1 \perp\!\!\!\perp X_2$$

f_{X_1} and f_{X_2} are **continuous**, **symmetric** and **unimodal** densities

$(\gamma^{\max}, \eta_1^{\text{mean}}, \eta_2^{\text{mean}})$ is a **globally optimal solution**



The optimal scheduling policy does not depend on the variance of the observations!

Sketch of proof

Lemma 1

The optimization problem can be cast in $\mathbb{R}^2$

$$\mathcal{J}(\gamma, \eta_1, \eta_2) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right] + c \cdot \mathbf{P}(U \neq 0)$$

For any given γ

$$\eta_i^*(y) = \begin{cases} x_i & \text{if} \quad y = x_i \\ \mathbf{E}[X_i \mid \gamma(X_1, X_2) \neq i] & \text{if} \quad y = \mathbf{S} \end{cases}$$

$\triangleq \hat{x}_i$ Representation point

Sketch of proof

For any given $(\hat{x}_1, \hat{x}_2) \in \mathbb{R}^2$

$$\gamma^*(x_1, x_2) = \begin{cases} 0 & \text{if } \; |x_1 - \hat{x}_1| \leq \sqrt{c}, |x_2 - \hat{x}_2| \leq \sqrt{c} \\ 1 & \text{if } \; |x_1 - \hat{x}_1| > \sqrt{c}, |x_1 - \hat{x}_1| \geq |x_2 - \hat{x}_2| \\ 2 & \text{otherwise} \end{cases}$$

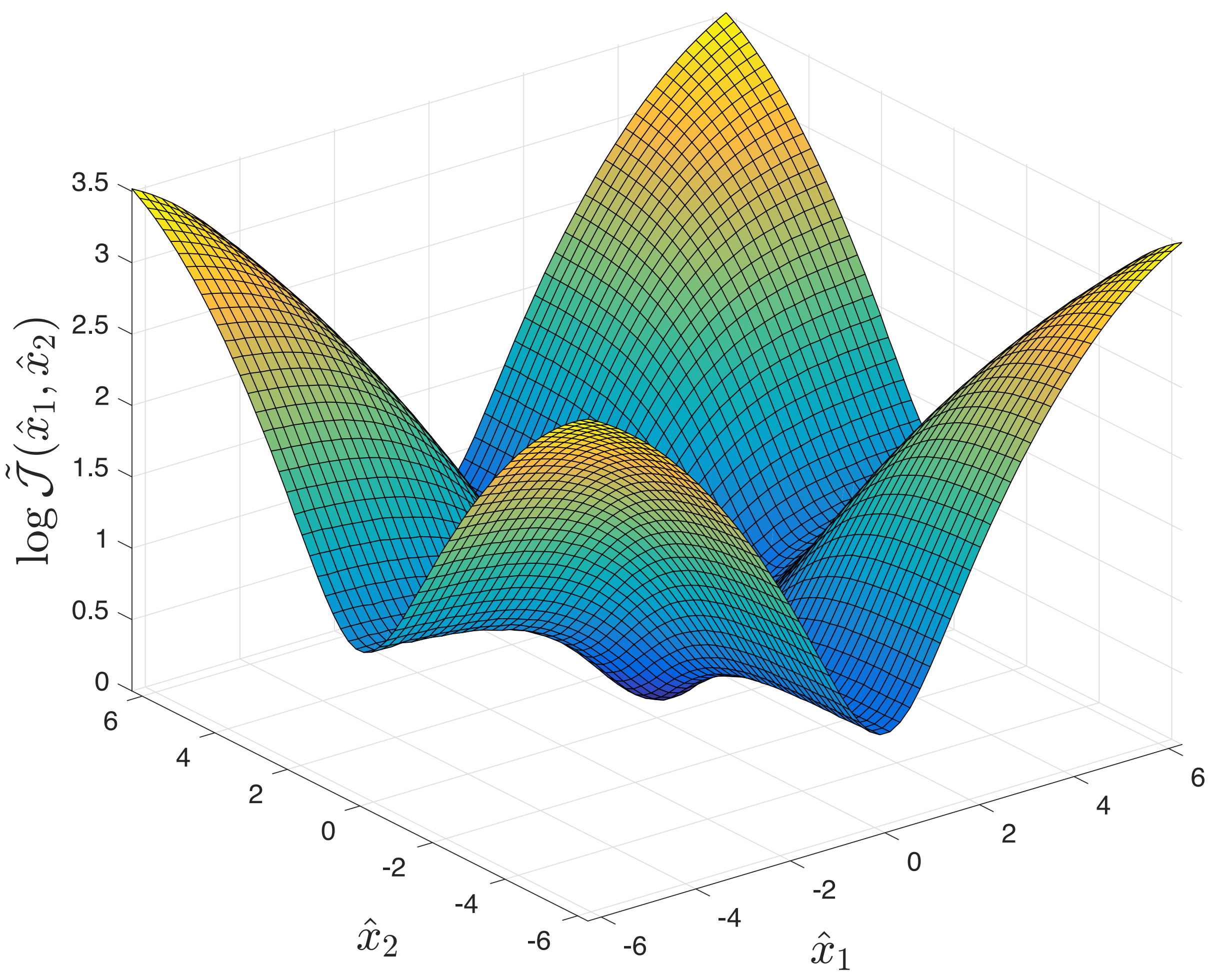
$$\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2) \triangleq \mathbf{E}\left[\min\left\{(X_1 - \hat{x}_1)^2 + (X_2 - \hat{x}_2)^2, (X_1 - \hat{x}_1)^2 + c, (X_2 - \hat{x}_2)^2 + c\right\}\right]$$

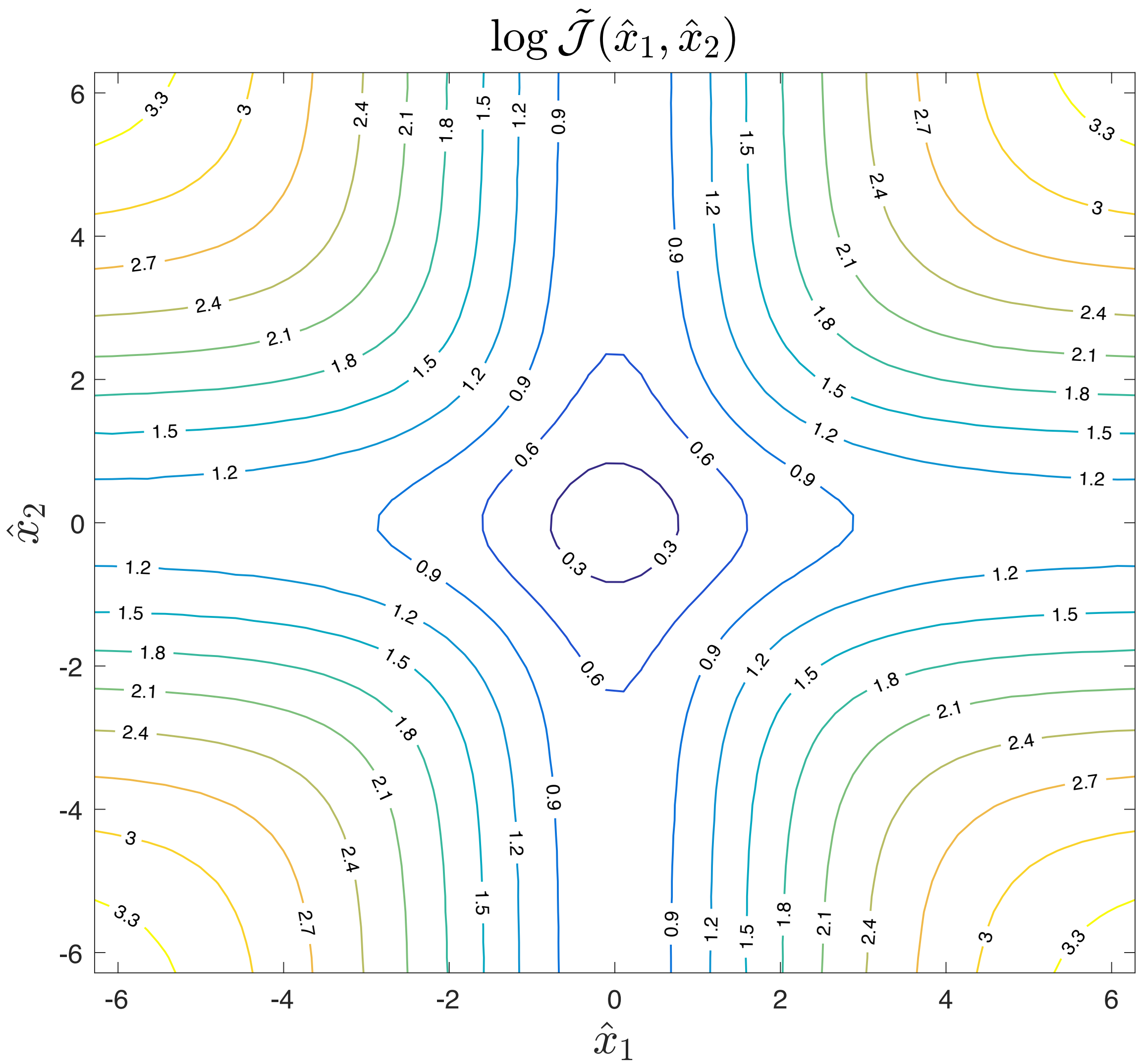
$$\underset{(\hat{x}_1, \hat{x}_2) \in \mathbb{R}^2}{\text{minimize}} \quad \tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2)$$

Finite dimensional cost function

$$X_1 \sim \mathcal{N}(0,1) \qquad X_2 \sim \mathcal{L}(0,2) \qquad c = 1$$

$$\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2) \triangleq \mathbf{E}\Big[\min\Big\{(X_1-\hat{x}_1)^2 + (X_2-\hat{x}_2)^2, (X_1-\hat{x}_1)^2 + c, (X_2-\hat{x}_2)^2 + c\Big\}\Big]$$





Nonconvex!

Sketch of proof

Lemma 2

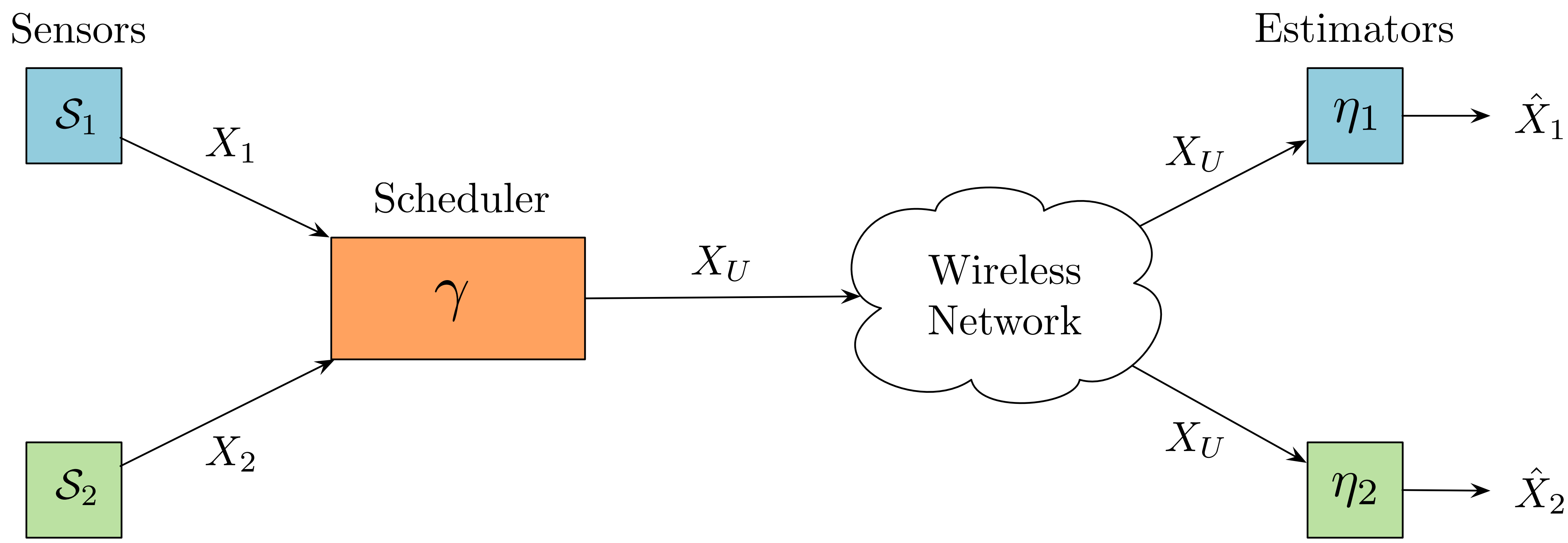
$(0,0)$ is a global minimizer of $\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2)$

$$\gamma^*(x_1, x_2) = \begin{cases} 0 & \text{if } \ |x_1 - \hat{x}_1| \le \sqrt{c}, |x_2 - \hat{x}_2| \le \sqrt{c} \\ 1 & \text{if } \ |x_1 - \hat{x}_1| > \sqrt{c}, |x_1 - \hat{x}_1| \ge |x_2 - \hat{x}_2| \\ 2 & \text{otherwise} \end{cases}$$

Max-scheduling policy

$$\gamma^{\max}(x_1, x_2) \triangleq \begin{cases} 0 & \text{if } \ |x_1|, |x_2| \le \sqrt{c} \\ \arg\max_i |x_i| & \text{otherwise} \end{cases}$$

Broadcast networks



$$\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2, \eta_1, \eta_2) \triangleq \mathbf{E}\Big[\min\Big\{(X_1 - \hat{x}_1)^2 + (X_2 - \hat{x}_2)^2, (X_1 - \eta_1(X_2))^2 + c, (X_2 - \eta_2(X_1))^2 + c\Big\}\Big]$$

Nonconvex, infinite dimensional!

Broadcast networks

Theorem

$$X_1 \perp\!\!\!\perp X_2$$

f_{X_1} and f_{X_2} are **continuous** and **symmetric** densities

$(\gamma^{\text{max}}, \eta_1^{\text{mean}}, \eta_2^{\text{mean}})$ is a **person-by-person optimal solution**

γ^{max} is optimal for $\eta_1^{\text{mean}}, \eta_2^{\text{mean}}$

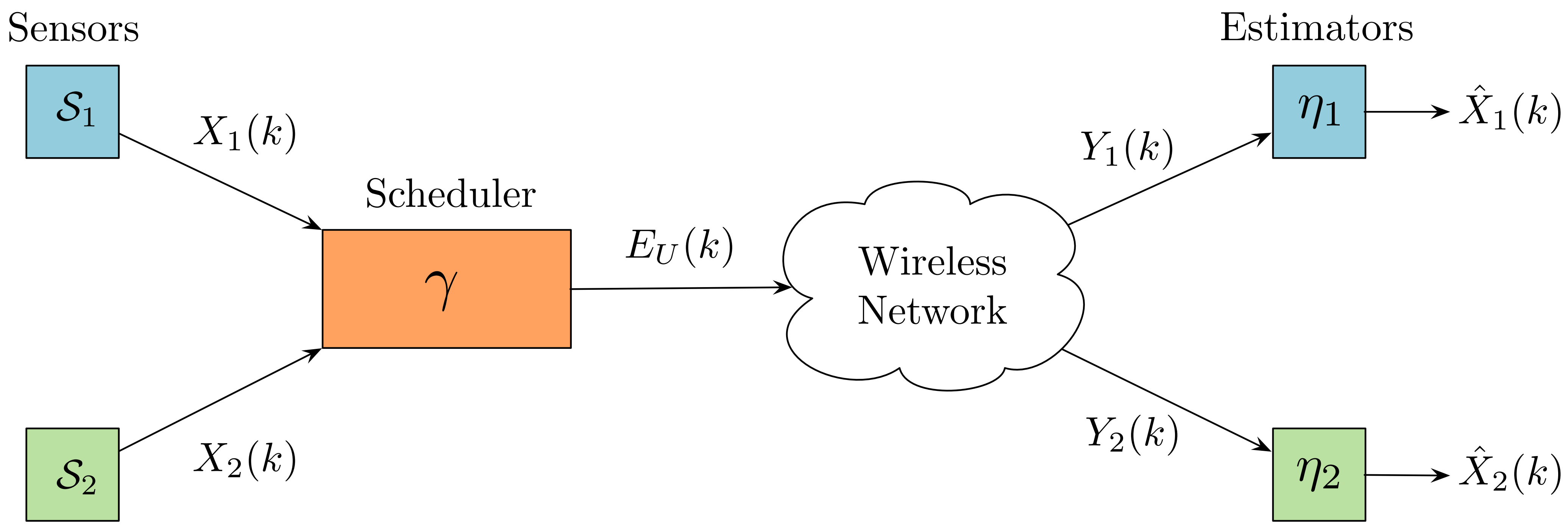
$\eta_1^{\text{mean}}, \eta_2^{\text{mean}}$ are optimal for γ^{max}

Necessary but not sufficient for global optimality

Application to linear systems

First-order linear systems

$$X_i(k+1) = A_i X_i(k) + W_i(k)$$

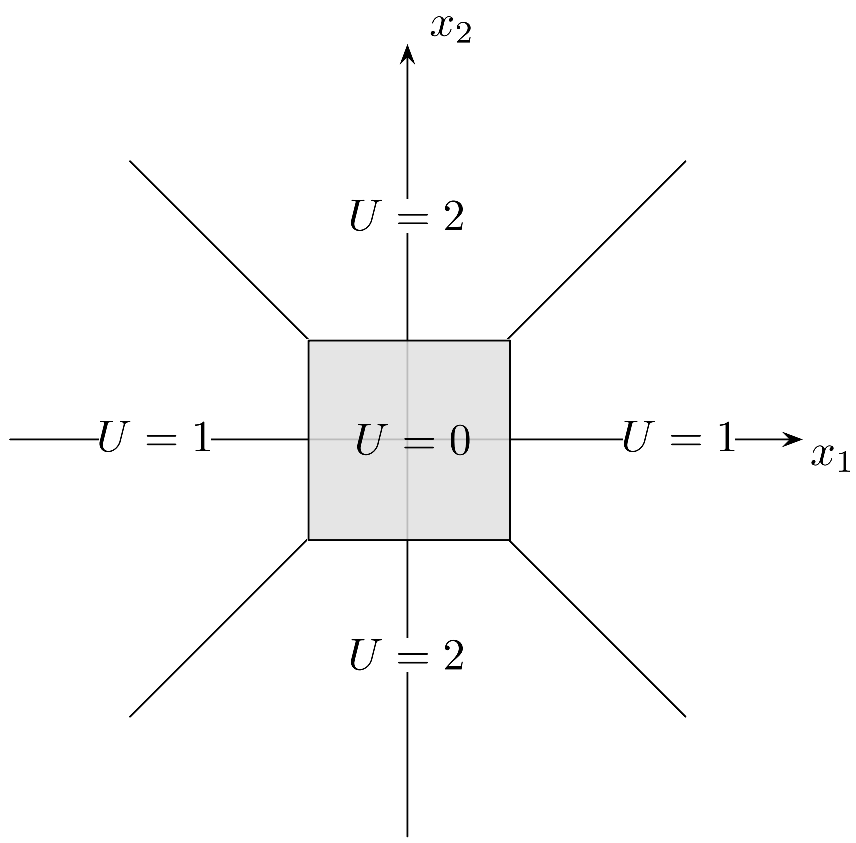


First-order linear systems

Innovation sequence

$$E_i(k) = X_i(k) - A_i \hat{X}_i(k-1)$$

max-scheduling



$$\hat{X}_i(k) = \begin{cases} A_i \hat{X}_i(k-1) & \text{if } \; Y_i(k) = \mathbf{S} \\ A_i \hat{X}_i(k-1) + E_i(k) & \text{if } \; Y_i(k) = \big(i, E_i(k)\big) \end{cases}$$

"Kalman-like" Estimator

Lipsa and Martins, TAC 2011 Nayyar et al., TAC 2013 Xia et al., TAC 2017 Xu and Hespanha, CDC 2005

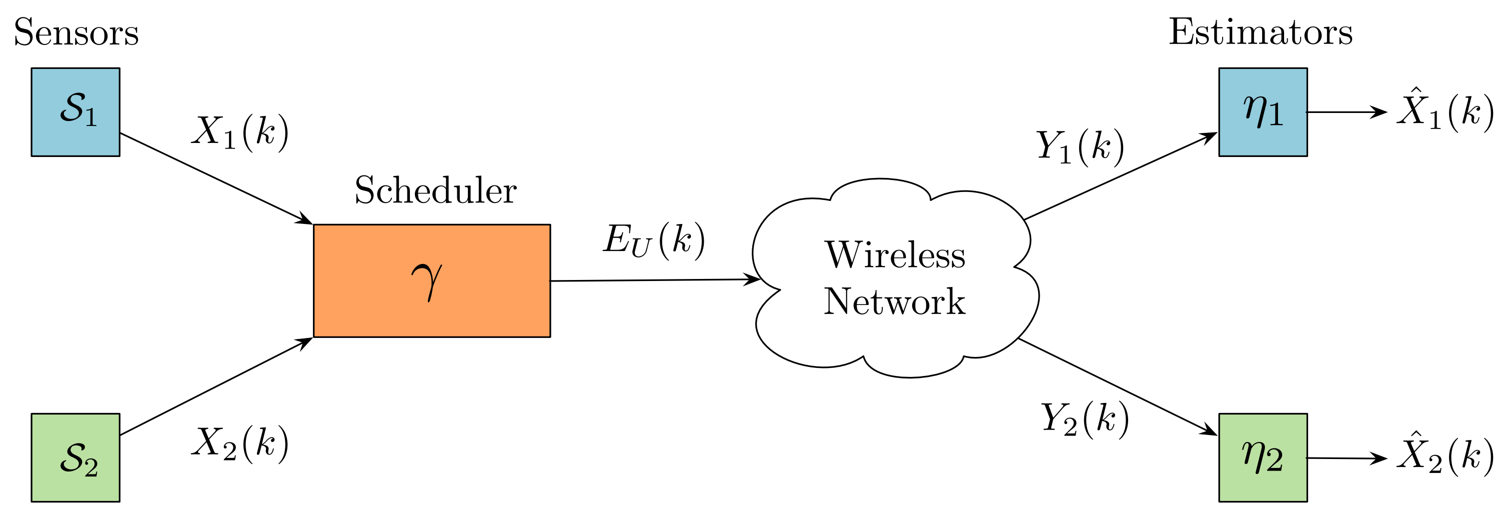
Remarks

1. **Optimal scheduling and estimation** strategies for iid state estimation
    - **Global optimality for unicast networks**
    - **Person-by-person optimality for broadcast networks**
2. Our results hold for **vectors** and **arbitrary number of sensors**
3. Application to the **scheduling of first order LTI systems**

Despite the **lack of convexity**, we found a **globally optimal solution**

Future work

- **Sequential problem formulations**:
  1. First order LTI with **aggregate error** cost
  2. IID sources with **limited number of transmissions**



$$\mathcal{J}(\gamma, \eta_1, \eta_2) = \sum_{k=1}^{T} \mathbf{E}\left[(X_1(k) - \hat{X}_1(k))^2 + (X_2(k) - \hat{X}_2(k))^2\right] + c \cdot \mathbf{P}(U(k) \neq 0)$$

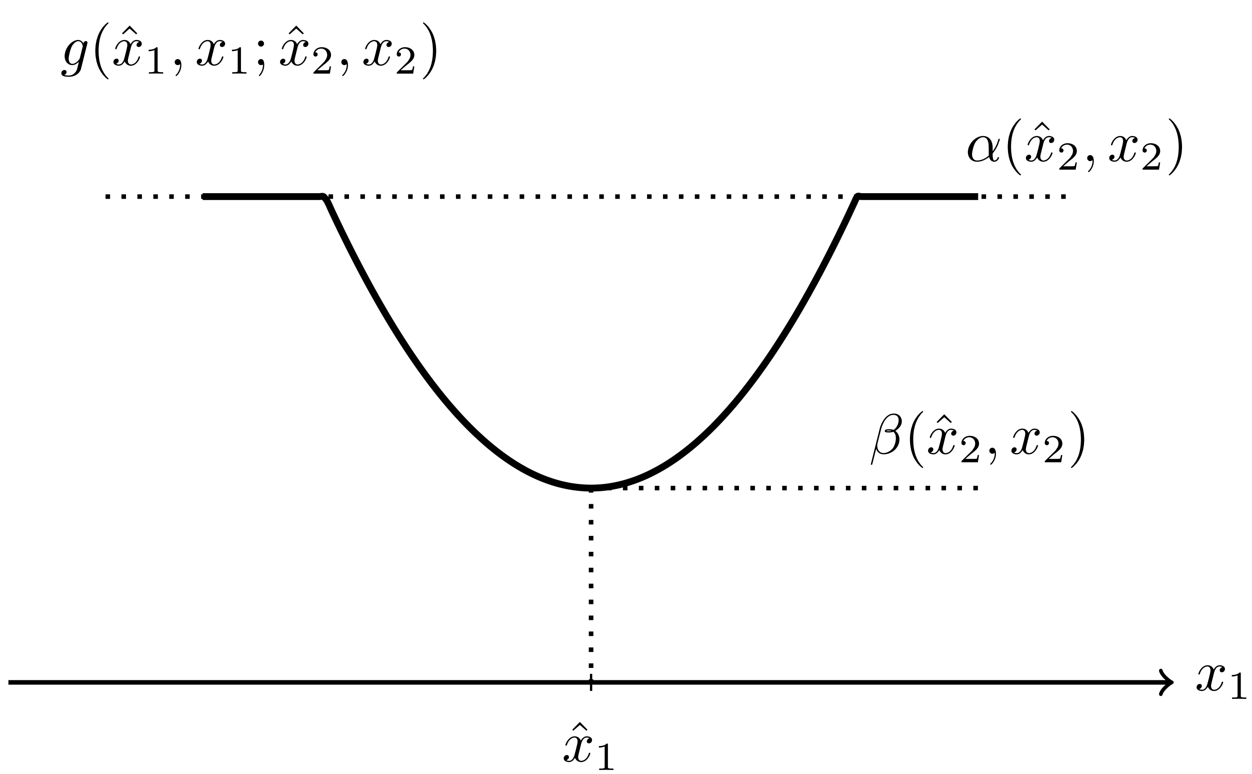
Appendix

Sketch of proof

Lemma

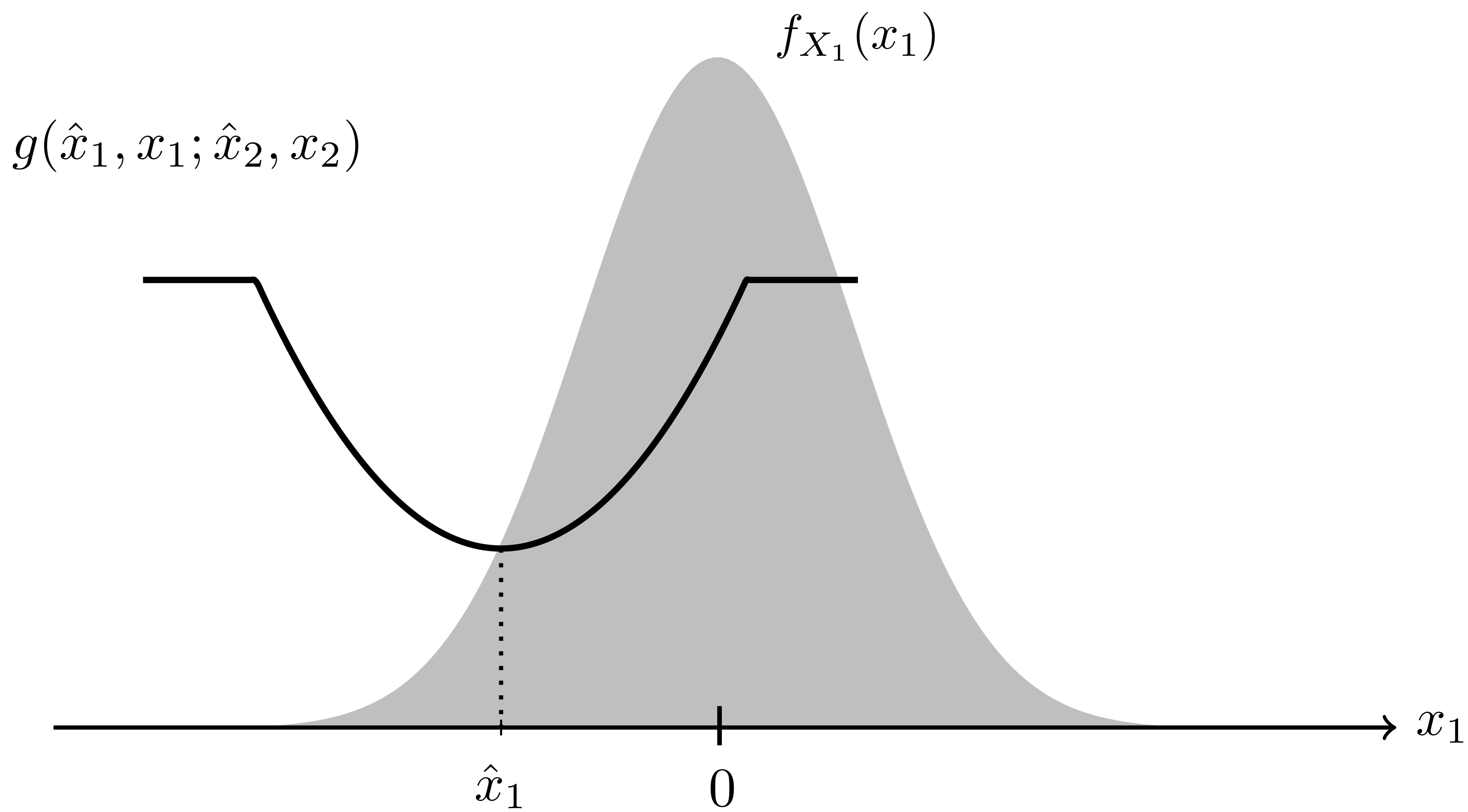
$$(0,0) \text{ is a global minimum of } \tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2)$$

$$\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2) = \int_{\mathbb{R}} \left[\int_{\mathbb{R}} g(\hat{x}_1, x_1; \hat{x}_2, x_2) f_{X_1}(x_1) dx_1 \right] f_{X_2}(x_2) dx_2$$



Sketch of proof

$$\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2) = \int_{\mathbb{R}} \left[\int_{\mathbb{R}} g(\hat{x}_1, x_1; \hat{x}_2, x_2) f_{X_1}(x_1) dx_1 \right] f_{X_2}(x_2) dx_2$$



$$\tilde{\mathcal{J}}(\hat{x}_1, \hat{x}_2) \geq \tilde{\mathcal{J}}(0, \hat{x}_2) \Longrightarrow \boldsymbol{\hat{x}_1^* = 0}$$

Follows from **Hardy-Littlewood** inequality