Observation-driven sensor scheduling

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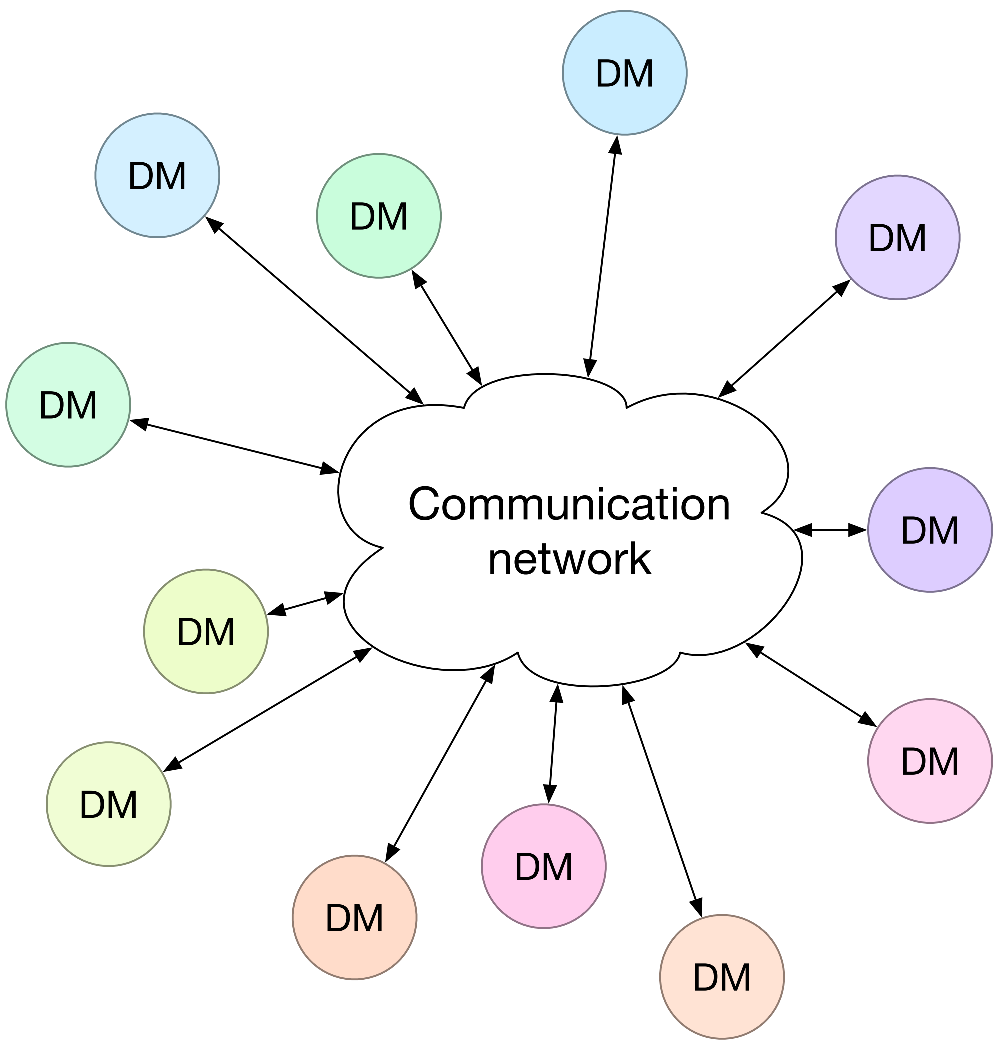
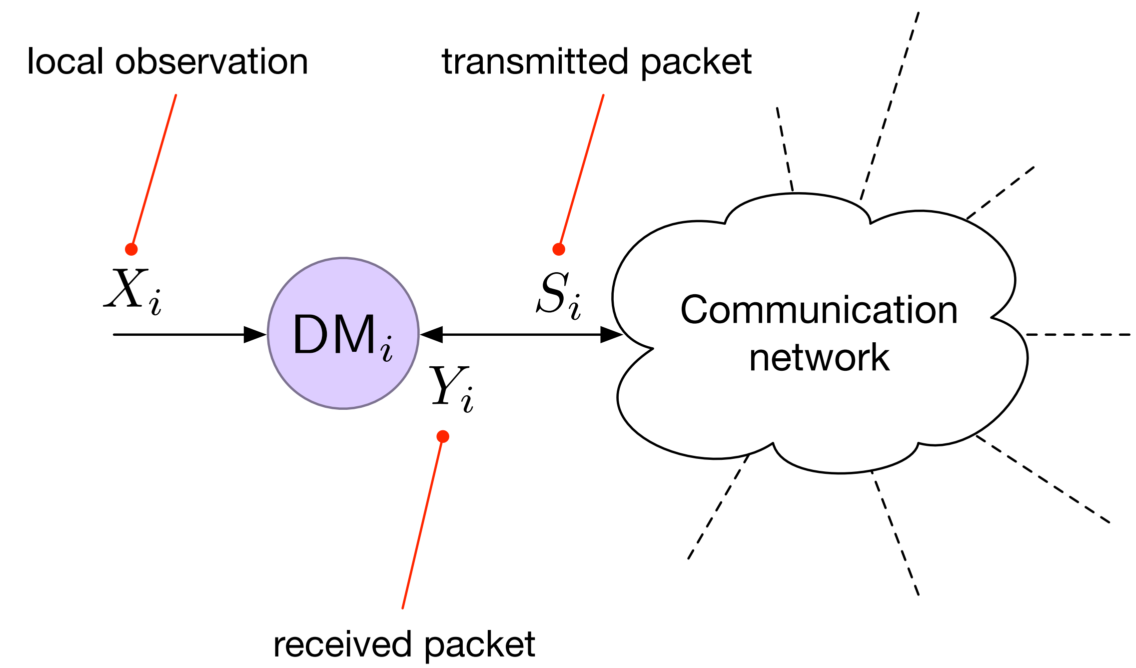
Networked decision systems





Many applications

1. **Networked control**
2. **Remote estimation**
3. **Sensor networks**
4. **Robotic networks**

Many challenges

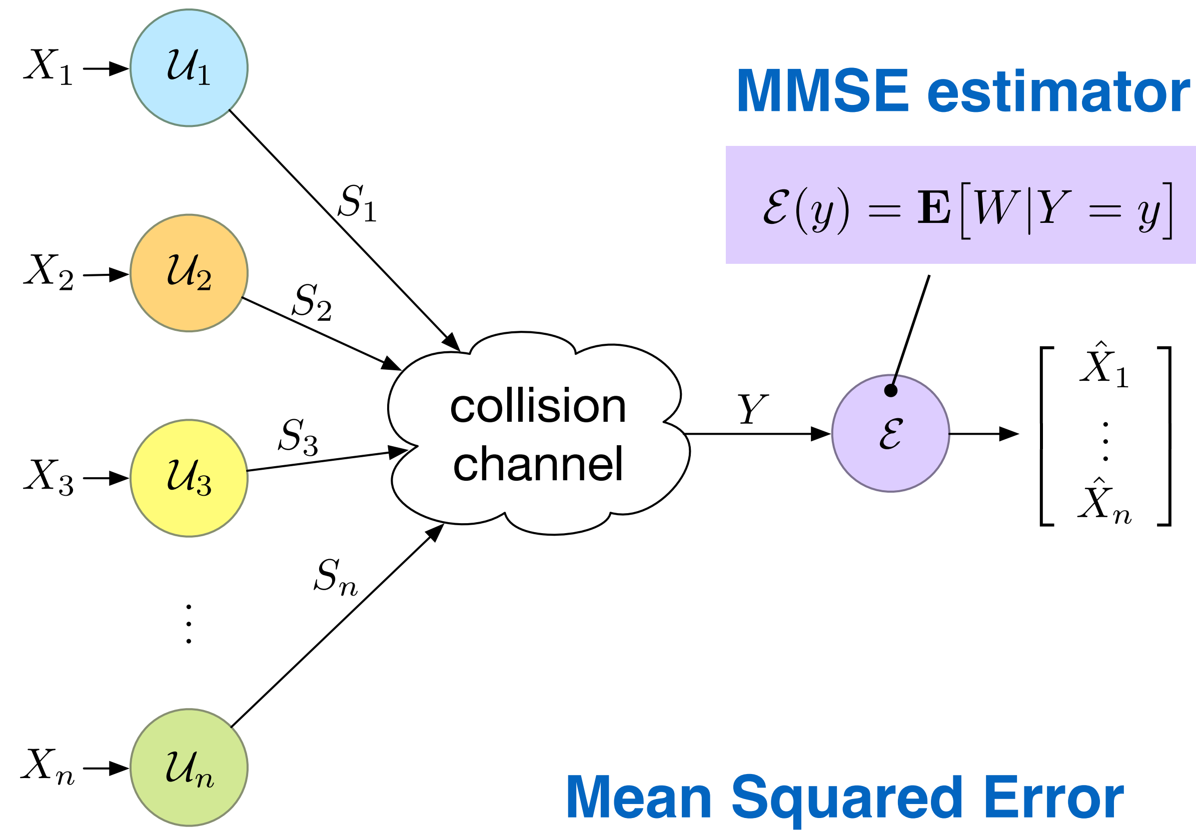
Communication is imperfect:
Delays, noise, quantization, congestion, packet drops, connectivity and **interference**

Previous work: MMSE estimation over the collision channel

$$W = [X_1, \cdots, X_n]$$

$$X_i, \quad i \in \{1, \cdots, n\}$$

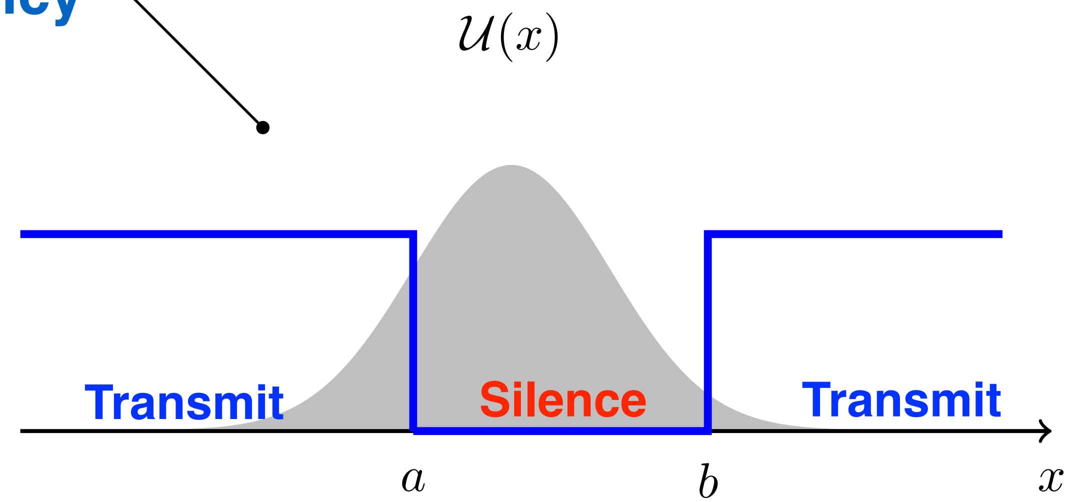
- mutually **independent**
- **continuous** rvs
- supported on the real line
- **any distribution**



Mean Squared Error

$$\text{minimize} \quad \mathcal{J}(\mathcal{U}_1, \ldots, \mathcal{U}_n) = \mathbf{E}\left[\sum_{i=1}^{n} (X_i - \hat{X}_i)^2\right]$$

Threshold policy



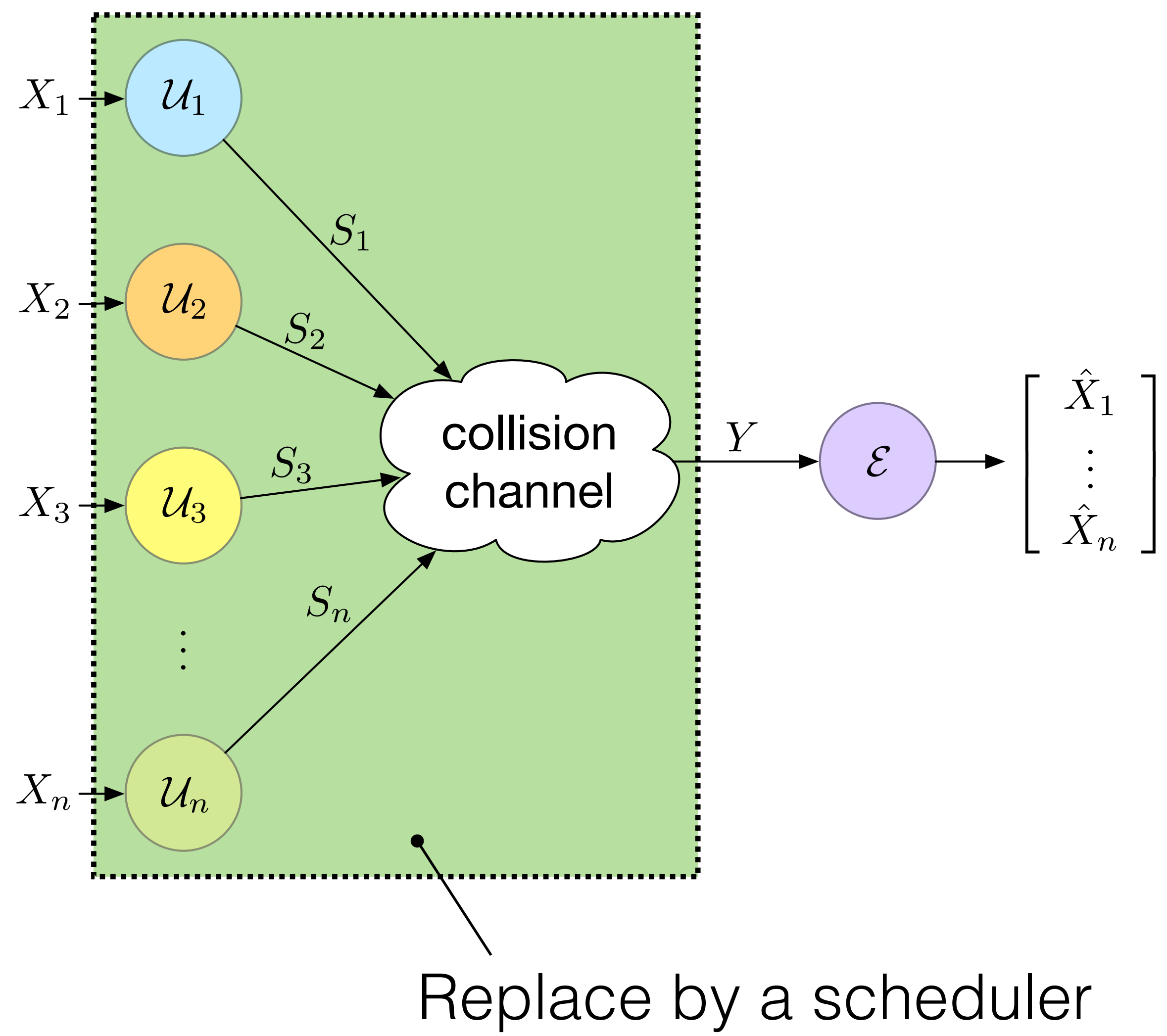
Result[1]

Existence of jointly optimal threshold policies

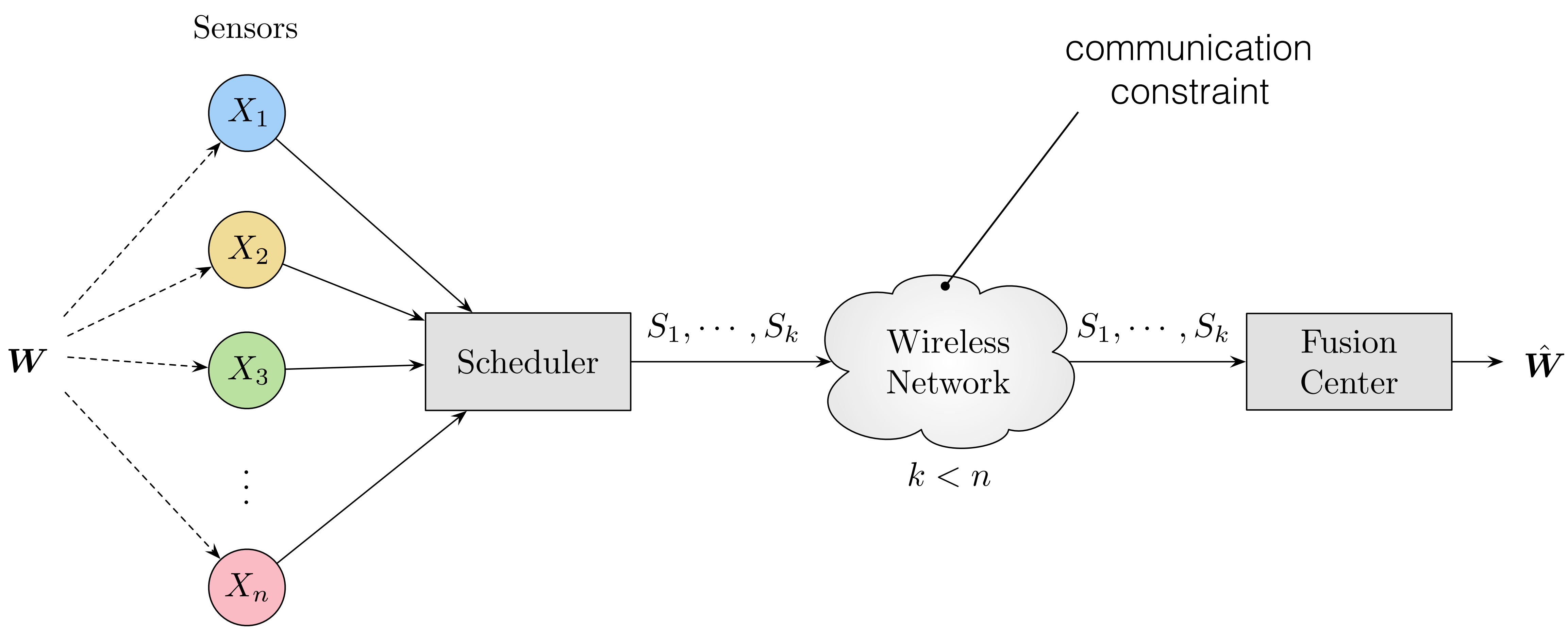
1. Vasconcelos & Martins - "Optimal estimation over the collision channel" *IEEE TAC* 2017

"Centralized" lower bound

$$\text{minimize} \quad \mathcal{J}(\mathcal{U}_1, \ldots, \mathcal{U}_n) = \mathbf{E}\left[\sum_{i=1}^{n}(X_i - \hat{X}_i)^2\right]$$



Basic framework

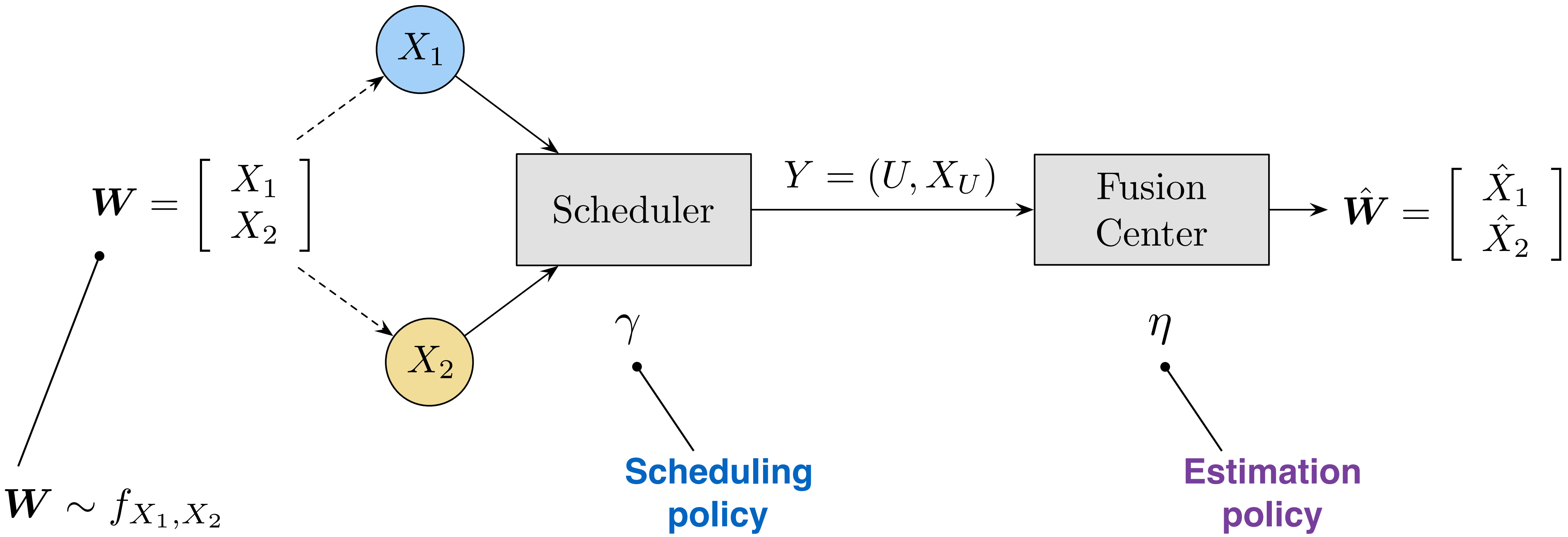


Sensor scheduling problem

Choose k out of n sensors such that
the expected distortion between $\boldsymbol{W}$ and $\hat{\boldsymbol{W}}$ is minimized

1. Athans - "On the determination of optimal costly measurement strategies for linear stochastic systems" *Automatica* 1972
2. Joshi and Boyd - "Sensor selection via convex optimization" *IEEE TSP* 2009

Simplest case: two sensors



Scheduling decision

$$U \in \{1, 2\}$$

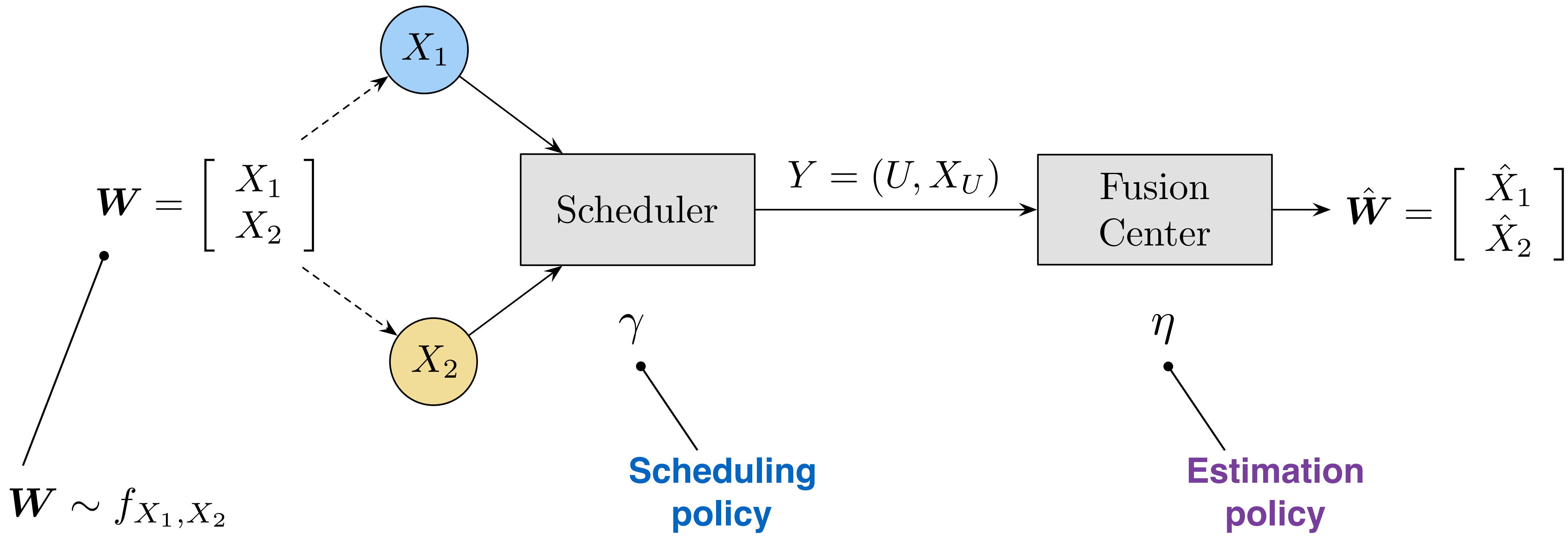
$$U = \gamma(X_1, X_2)$$

Estimate

$$\hat{W} = \eta(U, X_U)$$

The scheduler's decision **affects** what the fusion center observes

Simplest case: two sensors

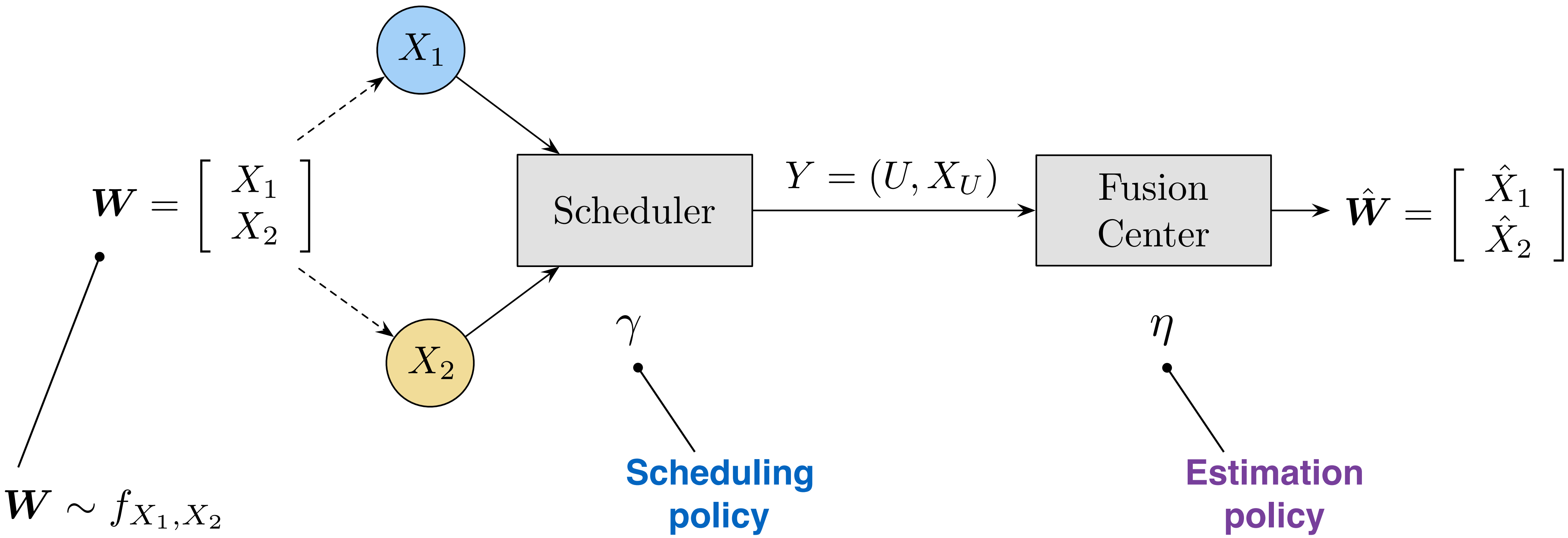


Find **scheduling** and **estimation** policies that **jointly** minimize the following cost

Team decision problem

$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta) = \mathbf{E}\Big[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\Big]$$

Simplest case: two sensors



$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right]$$

Open-loop scheduling: let the sensor with the **largest variance** transmit

Observation-driven scheduling[1]**:** let the sensor with the **"largest measurement"** transmit

1. Vasconcelos & Mitra - "Observation-driven sensor scheduling" *IEEE ICC* 2017

Notions of optimality

Team-optimality

$$\mathcal{J}(\gamma^*, \eta^*) \leq \mathcal{J}(\gamma, \eta), \quad (\gamma, \eta) \in \Gamma \times \mathrm{H}$$

$$\Longrightarrow$$

$$\not\Longleftarrow$$

Person-by-person optimality

$$\mathcal{J}(\gamma^*, \eta^*) \leq \mathcal{J}(\gamma, \eta^*), \quad \gamma \in \Gamma$$
$$\mathcal{J}(\gamma^*, \eta^*) \leq \mathcal{J}(\gamma^*, \eta), \quad \eta \in \mathrm{H}$$

$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma, \eta) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right]$$

Unfortunately, finding team-optimal solutions is **very difficult**

1. Witsenhausen - "A counterexample in optimal stochastic control" *SIAM J. Control* 1968
2. Tsitsiklis & Athans - "On the complexity of decentralized decision making and detection problems" *IEEE TAC* 1985

Notions of optimality

Team-optimality

$$\mathcal{J}(\gamma^*, \eta^*) \leq \mathcal{J}(\gamma, \eta), \quad (\gamma, \eta) \in \Gamma \times \mathrm{H}$$

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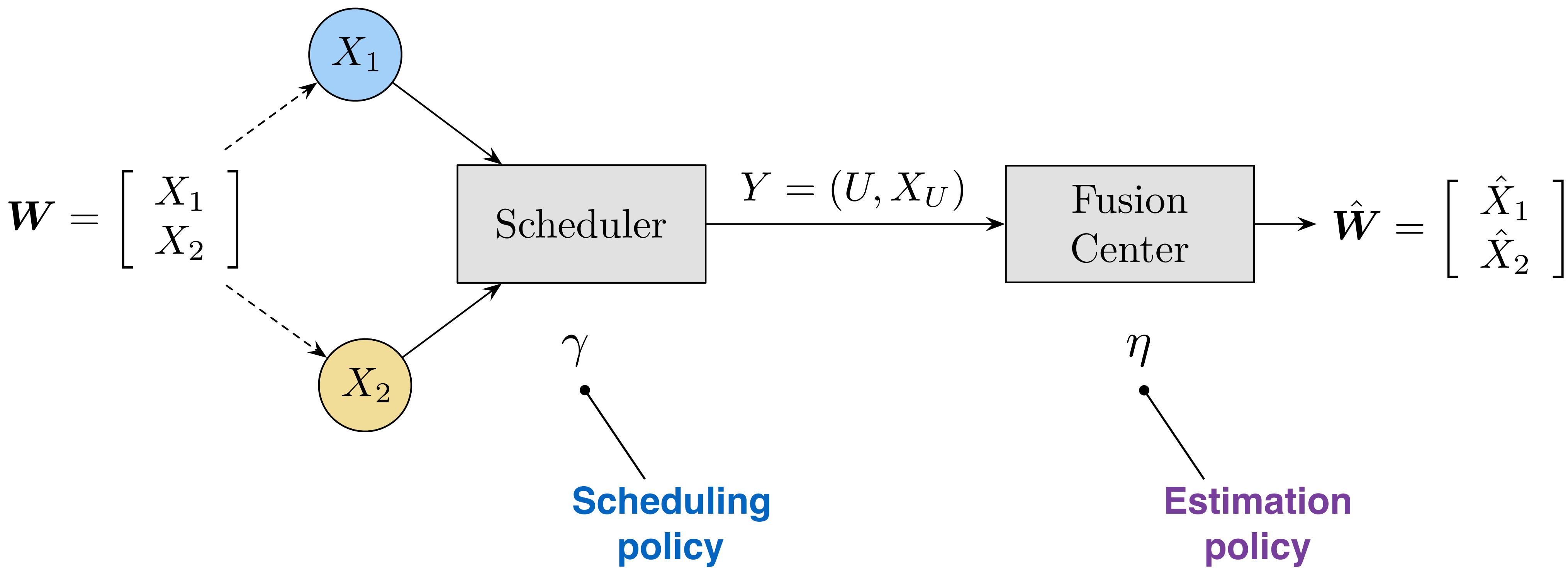
$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma, \eta) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right]$$

Finding person-by-person optimal solutions is **often much easier***

***depending on the probabilistic model of the source** f_{X_1,X_2}

Optimal estimation policy



Mean-squared Error

$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta) = \mathbf{E}\Big[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\Big]$$

MMSE estimator

$$\eta^*_\gamma(y) = \mathbf{E}\big[\boldsymbol{W} \mid Y = y\big]$$

The MMSE estimator is a **function** of the scheduling policy

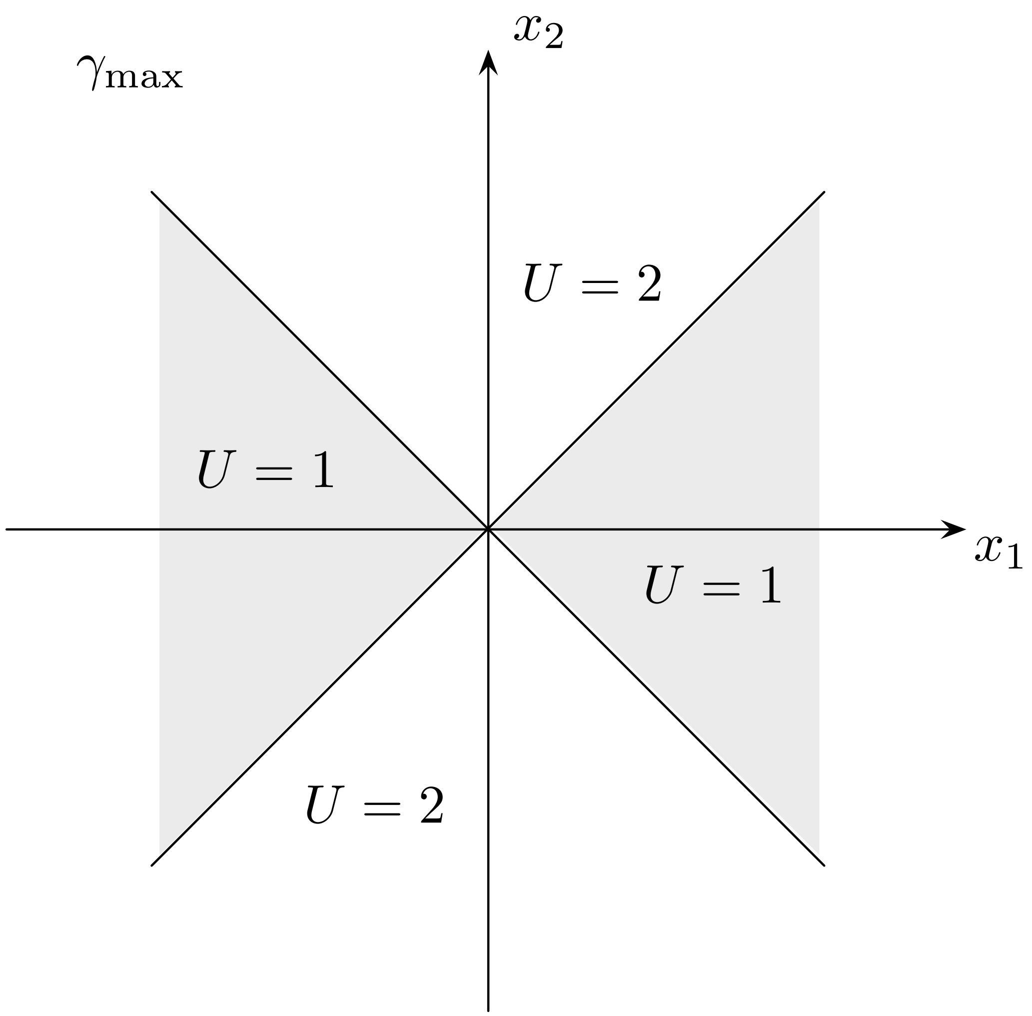
“Choose-the-max” scheduling

Theorem 1

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \right) \Longrightarrow \left(\gamma^{\max}, \eta^*_{\gamma^{\max}}\right) \text{ is person-by-person optimal}$$

“Choose-the-max” scheduling policy

$$\gamma^{\max}(x_1, x_2) = \begin{cases} 1, & \text{if } |x_1| \geq |x_2| \\ 2, & \text{otherwise} \end{cases}$$



Sketch of proof

The MMSE estimator for a given scheduling policy is

$$\eta^*_\gamma\big(\boldsymbol{Y} = (1, x_1)\big) = \begin{bmatrix} x_1 \\ \mathbf{E}\big[X_2 \mid U = 1, X_1 = x_1\big] \end{bmatrix}$$

$$\eta^*_\gamma\big(\boldsymbol{Y} = (2, x_2)\big) = \begin{bmatrix} \mathbf{E}\big[X_1 \mid U = 2, X_2 = x_2\big] \\ x_2 \end{bmatrix}$$

Suppose that

$$\gamma(x_1, x_2) = \begin{cases} 1, & \text{if } |x_1| \geq |x_2| \\ 2, & \text{otherwise} \end{cases}$$

then

$$\begin{aligned} \mathbf{E}\big[X_1 \,|\gamma(X_1, X_2) = 2, X_2 = x_2\big] &= \frac{\int_{\mathbb{R}} \tau \mathbf{1}(|\tau| < |x_2|) f_{X_1}(\tau) d\tau}{\int_{\mathbb{R}} \mathbf{1}(|\tau| < |x_2|) f_{X_1}(\tau) d\tau} \\ &= \frac{\int_{-|x_2|}^{|x_2|} \tau f_{X_1}(\tau) d\tau}{\int_{-|x_2|}^{|x_2|} f_{X_1}(\tau) d\tau} \equiv 0 \end{aligned}$$

Sketch of proof

Fix an estimation policy of the form:

$$\eta\big(\boldsymbol{Y} = (1, x_1)\big) = \begin{bmatrix} x_1 \\ \eta_2(x_1) \end{bmatrix} \qquad \eta\big(\boldsymbol{Y} = (2, x_2)\big) = \begin{bmatrix} \eta_1(x_2) \\ x_2 \end{bmatrix}$$

The cost becomes

$$\begin{aligned}\mathcal{J}(\gamma, \eta) = &\int_{\mathbb{R}^2} (x_2 - \eta_2(x_1))^2 \mathbf{1}(\gamma(x_1, x_2) = 1) f_{X_1, X_2}(x_1, x_2) dx_1 dx_2 \\ &+ \int_{\mathbb{R}^2} (x_1 - \eta_1(x_2))^2 \mathbf{1}(\gamma(x_1, x_2) = 2) f_{X_1, X_2}(x_1, x_2) dx_1 dx_2\end{aligned}$$

[Generalized] Nearest Neighbor Condition:

$$\gamma^*(x_1, x_2) = 1 \Leftrightarrow \big(x_1 - \eta_1(x_2)\big)^2 \geq \big(x_2 - \eta_2(x_1)\big)^2$$

Sketch of proof

[Generalized] Nearest Neighbor Condition:

$$\gamma^*(x_1, x_2) = 1 \Leftrightarrow \big(x_1 - \eta_1(x_2)\big)^2 \geq \big(x_2 - \eta_2(x_1)\big)^2$$

Suppose that $\eta_1(x_2) = \eta_2(x_1) \equiv 0$

then $$\gamma^*(x_1, x_2) = 1 \Leftrightarrow \big(x_1 - 0\big)^2 \geq \big(x_2 - 0\big)^2$$
$$|x_1| \geq |x_2|$$

■

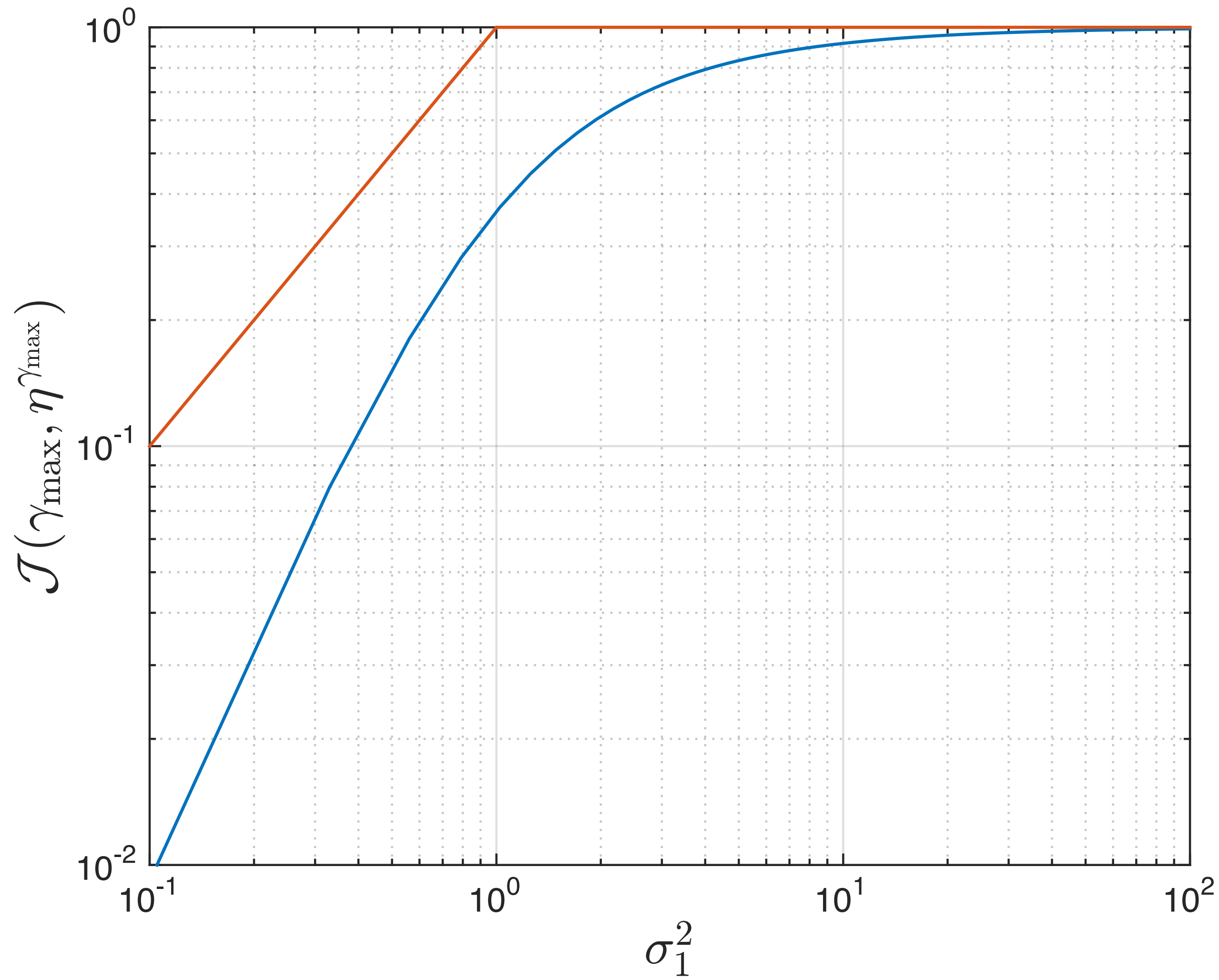
Performance of "choose-the-max"

$$\mathcal{J}(\gamma_{\max}, \eta^{\gamma_{\max}}) = \mathbf{E}\Big[\min\{X_1^2, X_2^2\}\Big]$$

Observation-driven sensor scheduling

$$\bar{\mathcal{J}}(\sigma_1^2, \sigma_2^2) = \min\{\sigma_1^2, \sigma_2^2\}$$

"Open-loop" sensor scheduling



Remarks

1. Result only depends on the even symmetry of the density
2. Can be extended to **any number of sensors** making **vector observations**[1]

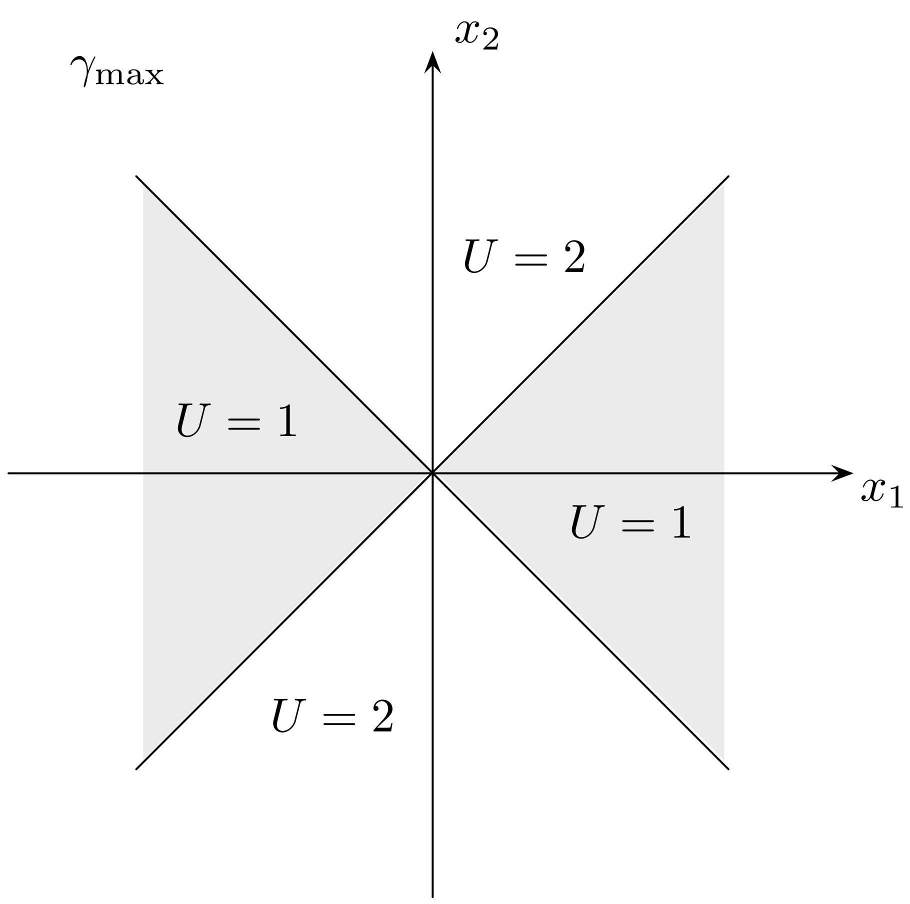
1. Vasconcelos, Nayyar & Mitra - "Optimal scheduling strategies in networked estimation" submitted to *IEEE CDC*

Symmetrically correlated case

Theorem 2

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right) \implies (\gamma^{\max}, \eta^*_{\gamma^{\max}}) \text{ is person-by-person optimal}$$

$$X_i \mid X_j = x \sim \mathcal{N}(\rho \cdot x, 1 - \rho^2)$$

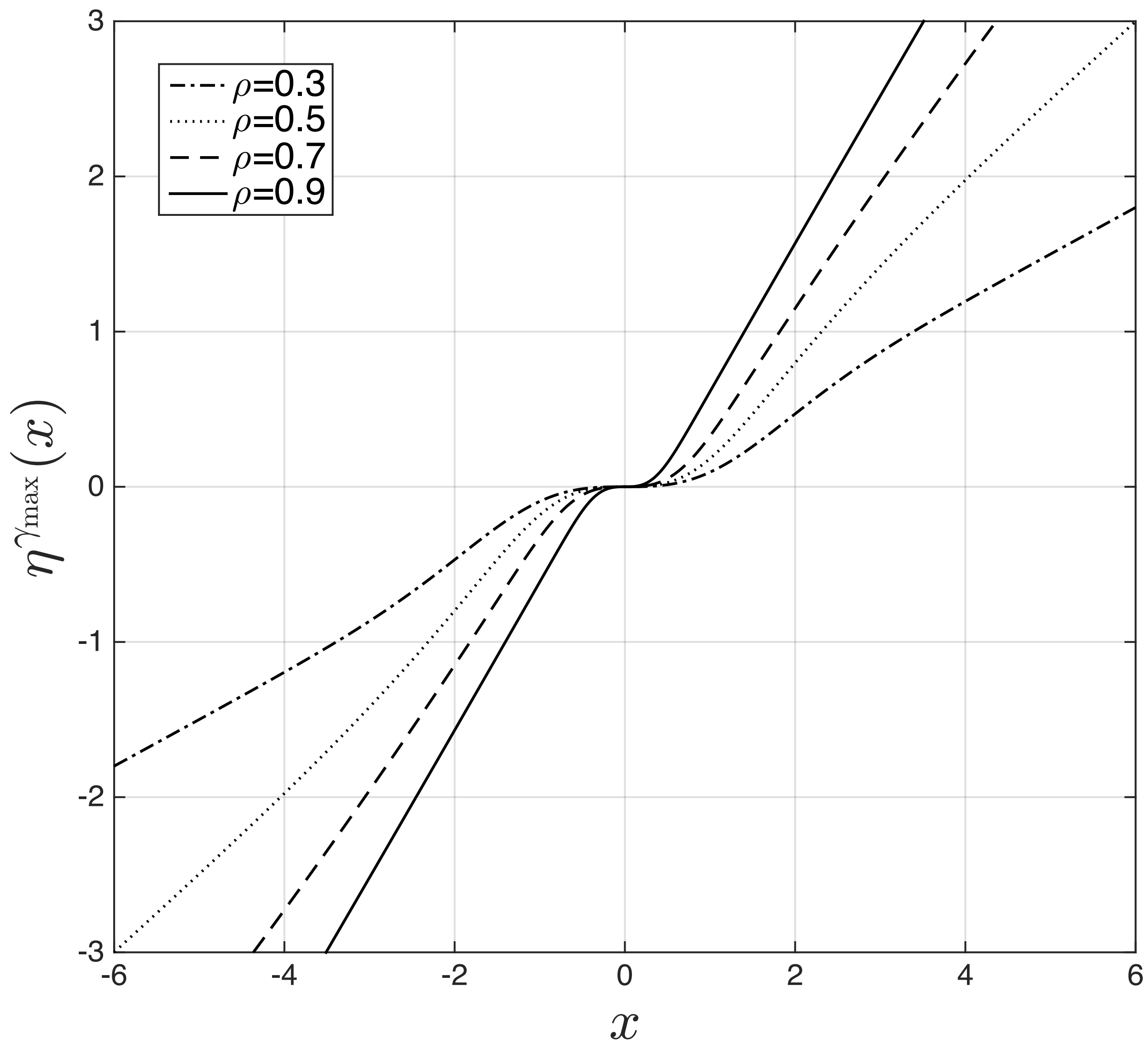


$$\eta^*_{\gamma_{\max},i}(x) = \frac{\int_{-|x|}^{|x|} \tau f_{X_i|X_j=x}(\tau)d\tau}{\int_{-|x|}^{|x|} f_{X_i|X_j=x}(\tau)d\tau}$$

Optimal nonlinear estimator

$$\eta^*_{\gamma_{\max},i}(x) = \frac{\int_{-|x|}^{|x|} \tau f_{X_i|X_j=x}(\tau) d\tau}{\int_{-|x|}^{|x|} f_{X_i|X_j=x}(\tau) d\tau}$$

$$f_{X_i|X_j=x}(\tau) = \frac{1}{\sqrt{2\pi(1-\rho^2)}} \exp\left[-\frac{(\tau-\rho\cdot x)^2}{2(1-\rho^2)}\right]$$



Symmetry properties

1.

$$\eta^*_{\gamma_{\max},1} = \eta^*_{\gamma_{\max},2}$$

2.

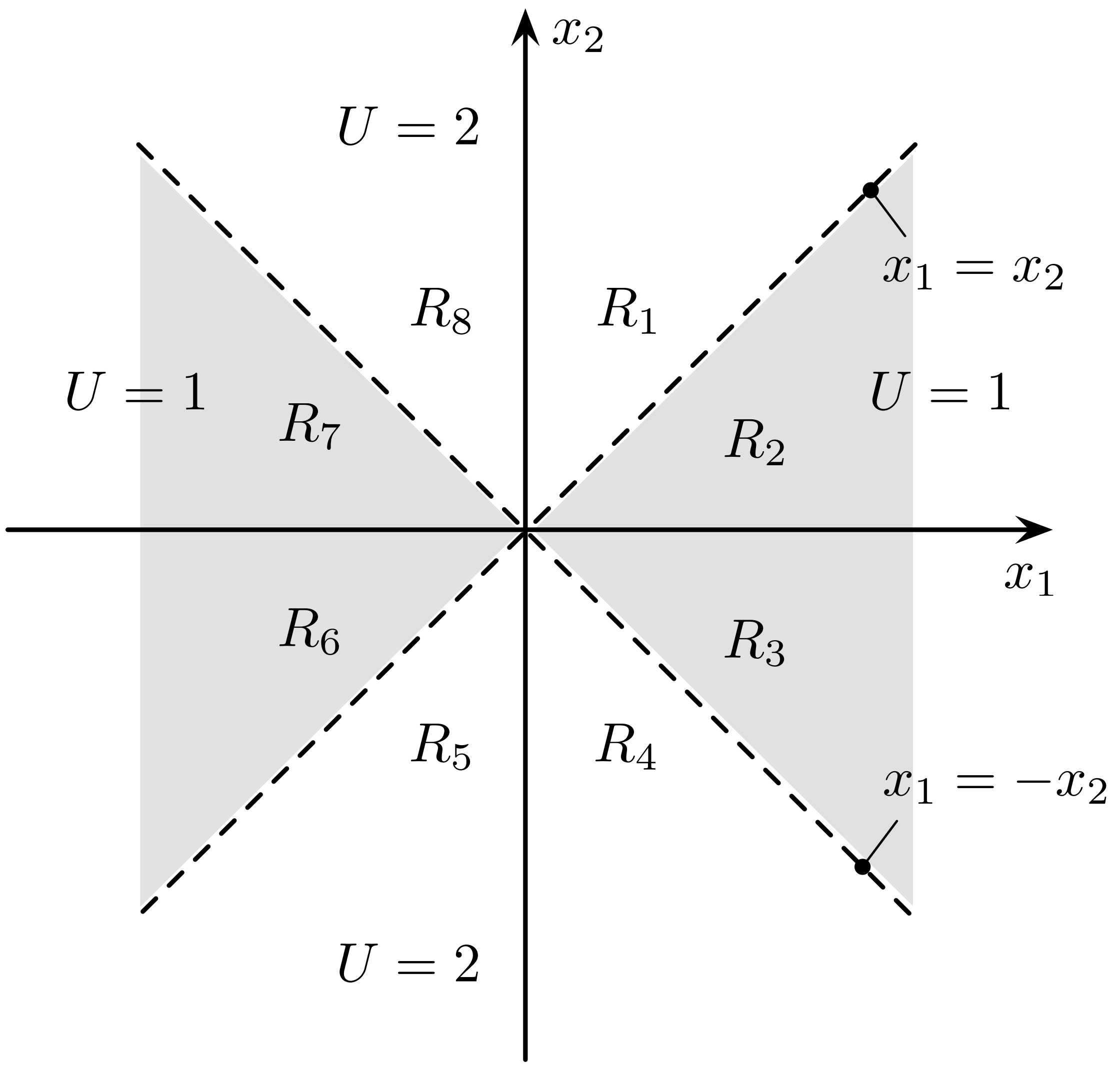
$$\eta^*_{\gamma_{\max}}(-x) = -\eta^*_{\gamma_{\max}}(x)$$

Sketch of proof

[Generalized] Nearest Neighbor Condition:

$$\gamma^*(x_1, x_2) = 1 \Leftrightarrow \left(x_1 - \eta_{\gamma_{\max}}(x_2)\right)^2 - \left(x_2 - \eta_{\gamma_{\max}}(x_1)\right)^2 \geq 0$$

$$H(x_1, x_2) \geq 0$$



Show that

$$H(x_1, x_2) \geq 0, \quad (x_1, x_2) \in R_2 \cup R_3 \cup R_6 \cup R_7$$

$$H(x_1, x_2) \leq 0, \quad (x_1, x_2) \in R_1 \cup R_4 \cup R_5 \cup R_8$$

Sketch of proof

Fact 1

$$H(x_1, x_2) = \big(F(x_1) + F(x_2)\big) \times \big(G(x_1) - G(x_2)\big)$$

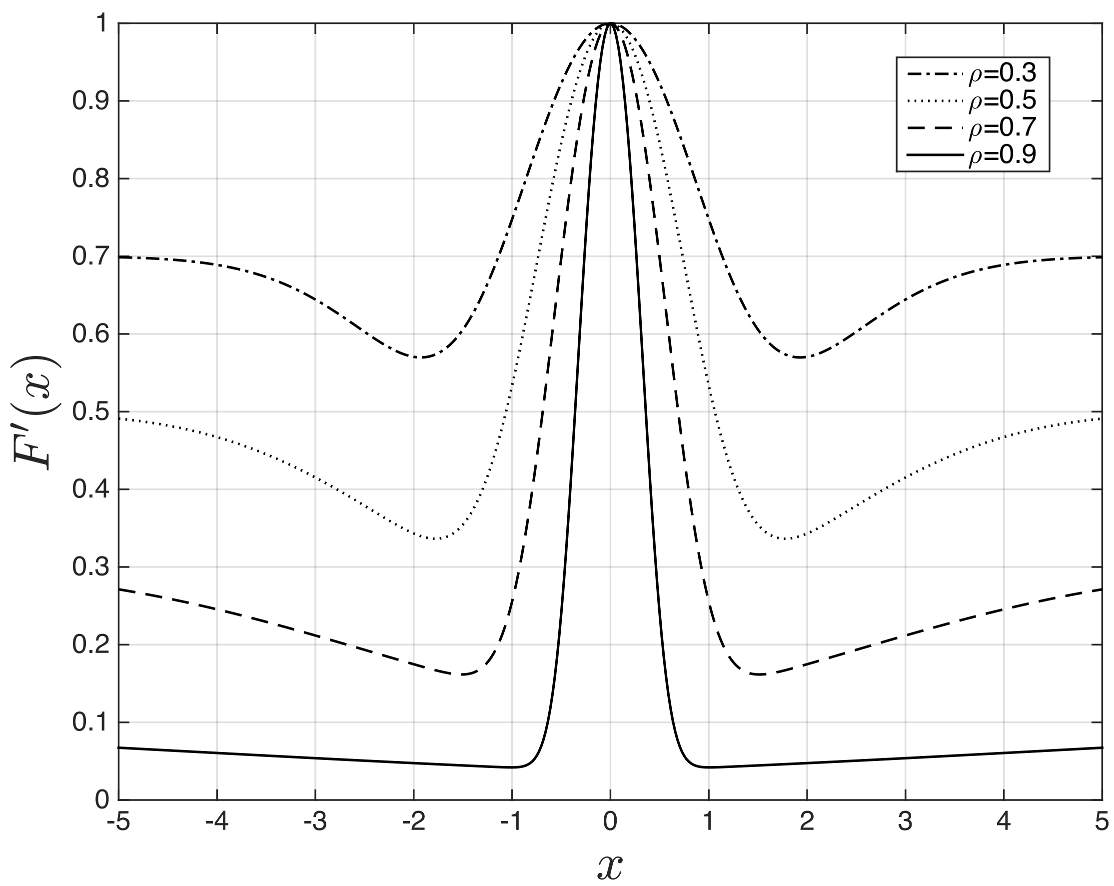
$$F(x) = x - \eta^*_{\gamma_{\max}}(x) \qquad G(x) = x + \eta^*_{\gamma_{\max}}(x)$$

Fact 2

F and G are odd functions

Fact 3*

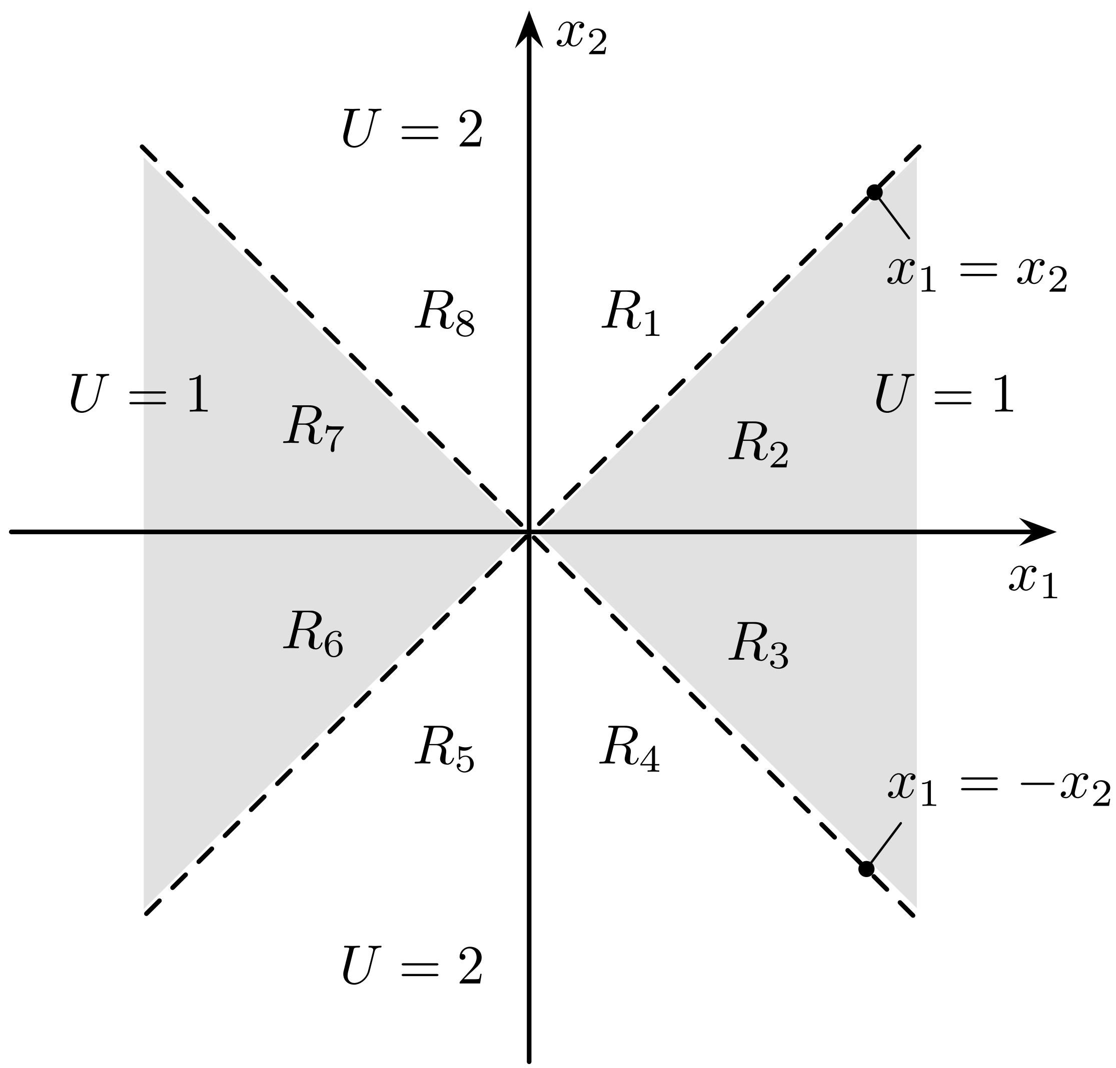
F and G are monotone increasing



***still need an analytical proof…**

Sketch of proof

$$H(x_1, x_2) = \big(F(x_1) + F(x_2)\big) \times \big(G(x_1) - G(x_2)\big)$$



Let $(x_1, x_2) \in R_1$

Then

$$x_1 \geq 0 \Rightarrow F(x_1) \geq F(0) = 0$$
$$x_2 \geq 0 \Rightarrow F(x_2) \geq F(0) = 0$$

(increasing; odd)

and

$$x_2 \geq x_1 \Rightarrow G(x_2) \geq G(x_1)$$

(increasing)

Therefore

$$H(x_1, x_2) \leq 0$$

∎

Performance of “choose-the-max”

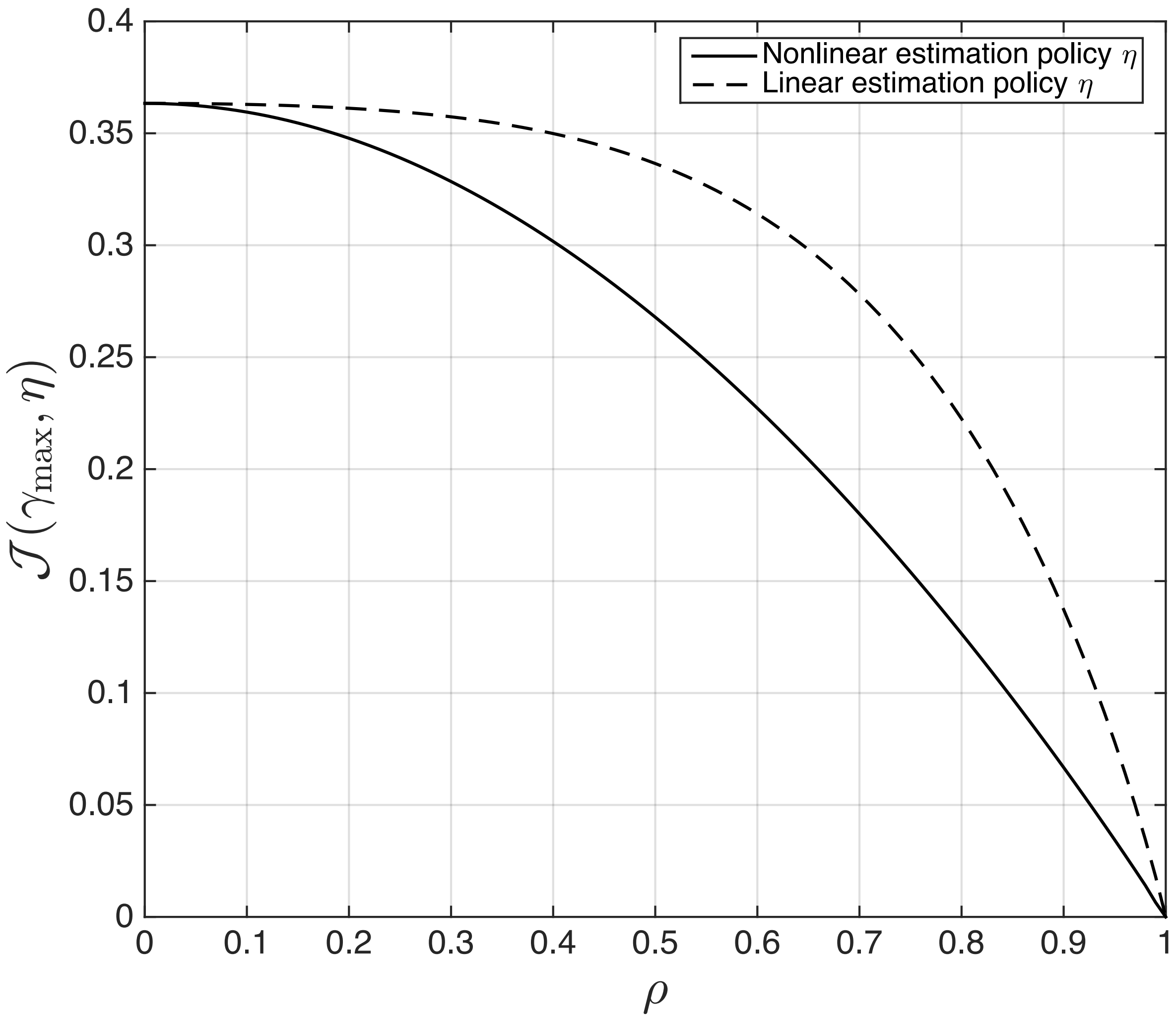
$$\gamma^{\max}(x_1, x_2) = \begin{cases} 1, & \text{if } |x_1| \geq |x_2| \\ 2, & \text{otherwise} \end{cases}$$

MMSE estimator

$$\eta^*_{\gamma_{\max},i}(x) = \frac{\int_{-|x|}^{|x|} \tau f_{X_i|X_j=x}(\tau)d\tau}{\int_{-|x|}^{|x|} f_{X_i|X_j=x}(\tau)d\tau}$$

LMMSE estimator

$$\eta_i(x) = \rho \cdot x$$



1. Both estimators induce the “choose-the-max” policy
2. Computational complexity vs. performance trade-off

Conclusions

1. Introduced a **new approach** to sensor scheduling
2. Person-by-person optimality of **“choose-the-max”** scheduling policy and its corresponding **conditional mean** estimator
3. A **nontrivial lower bound** to estimation over the collision channel

Future work

1. Extend these results to the **general multivariate Gaussian** case
2. Establish a connection with **compressive sensing**
3. **Sequential** problem formulations (much more involved)

Open question

Can we establish global optimality?

Scenes from the next "episode"

Generalized nearest neighbor condition

$$\gamma^*(x_1, x_2) = 1 \Leftrightarrow \big(x_1 - \eta_1(x_2)\big)^2 \geq \big(x_2 - \eta_2(x_1)\big)^2$$

Infinite dimensional optimization

$$\mathcal{J}(\gamma^*_\eta, \eta) = \mathbf{E}\left[\min\left\{\big(X_1 - \eta_1(X_2)\big)^2, \big(X_2 - \eta_2(X_1)\big)^2\right\}\right]$$

Constrain to affine estimators

$$\mathcal{J}(a, b, c, d) = \mathbf{E}\left[\min\left\{\big(X_1 - aX_2 - b\big)^2, \big(X_2 - cX_1 - d\big)^2\right\}\right]$$

Nonconvex
Difference of convex
Convex-concave procedure
Approximate subgradient

Networked control systems